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Leverage and risky investments

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LEVERAGE AND RISKY INVESTMENTS

by

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INTRODUCTION

Purpose of the Study

The purpose of this thesis is to construct a model that will demonstrate that a higher investment indifference curve can be reached (higher level of satisfaction) in terms of a combination of risk and expected rate of return when more debt is used, assuming diversification.

The Problem

The methods of corporate financing are traditionally classified into two major groups: one is equity financing and the other is debt financing. The rationale for this classification is that the nature of funds are different in two main respects. First, the interest payment on debt is fixed by contract, whereas dividend payments represent a variable cost which can vary freely in accordance with the performance of the business or enterprise. Second, interest payments on debt are deductible for corporate income tax purposes, whereas dividends are not.[1]

According to Solomon there are three problems to which financial management should direct attention.[2]

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[1]This difference is institutional and political rather than economic. In the economic sense, capital has a cost to the firm whether it is supplied by owners or lenders. In the long run, suppliers of equity capital must be paid the "normal" rate of return either in the form of dividends or capital gains.

[2]Ezra Solomon, The Theory of Financial Management (New York & London: Columbia University Press, 1963) p. 8.

1. What specific assets should an enterprise acquire?
2. What total volume of funds should an enterprise commit to the acquisition of such assets?
3. How should the necessary funds be acquired?

The central problem of concern in this thesis is the latter. What insight can be gained regarding the "ideal" debt-equity ratio? Two main arguments regarding these questions have been advanced:

1. One of the traditional arguments is that the use of more debt accrues earnings to equity capital, and that the increased risk caused by using debt may not be reflected in stock prices. In this situation, the market is willing to buy more of the corporation's common stock at a higher level of risk.[3]

2. A recent argument advanced by Modigliani and Miller is that the use of more debt causes investors to require compensation for the additional risk. In this case, an increase in the debt-equity ratio results in an increase in the cost of equity capital. The decreased cost of capital by using debt is presumed to be offset by the increased cost of equity capital.[4]

3. A still more recent and generally accepted argument is that as the percentage of the debt in total financing exceeds a certain level, the financial risk also increases. As the lenders' risk increases

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[3]Arthur Stone Dewing, <u>The Financial Policy of Corporations</u> (5th ed; New York: The Ronal Press Co., 1953) pp. 836-843.

[4]Franco Modigliani and M. H. Miller, <u>The Cost of Capital, Corporation Finance, and the Theory of Investment</u> (AER, XLVII, June 1958) pp. 261-297.

and lenders impose higher rates of interest, the total cost of capital increases. Under this thesis there is some optimum point of debt equity ratio. Beyond that point the total cost of capital increases rapidly.[5]

This thesis deals with the relationship between debt financing and risk and expected rate of return under the diversification situation. This writer does not use the cost of capital approach in which the risk factor and the earning power factor are combined to determine the capitalization rate. Rather, risk and earning power are separated, evaluated and compared under alternative financing situations--equity financing and debt financing. This approach attempts to locate maximum investment utility, assuming the indifference curve represents various combinations of risk and expected rate of return which are equally satisfactory to the firm.

Definitions

Risk and Uncertainty. It is necessary to distinguish clearly between risk and uncertainty. When a set of alternative future outcomes can be assigned a definite probability distribution with confidence, the outcome situation is called "risk." When no specific probability distribution can be assigned with confidence, the outcome situation is called "uncertainty."[6] This distinction came originally from Professor Frank H. Knight. Professor Knight says:

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[5] Ezra Solomon, *op. cit.*, pp. 91-106.

[6] Alexander A. Robichek and Steward C. Mayers, *Optimal Financing Decisions* (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1965) pp. 16-17.

> It will appear that a measurable uncertainty, or risk proper, as we shall use the term, is so far different from an unmeasurable one that it is not in effect an unmeasurable one, that it is not in effect an uncertainty at all. We shall accordingly restrict the term "uncertainty" to cases of the non-quantitative type.[7]

Leverage and Diversification. It is assumed in the following discussion that each unit of investment is of equal size and that the outcome of each investment is independent of the other outcomes. Therefore, when the total amount of investments is increased by using debt, two different types of effects are automatically in the total effect; that is, the "leverage effect" and the "diversification effect." The "leverage effect" is that part of the total effect which results from the change in the debt-equity ratio, and the "diversification effect" is that part of the total effect which results from the change in the investment level *per se*. When debt and equity financing are compared under the same investment situation, the *similarity* should be attributed to the diversification effect and the *difference* between them should be attributed to the leverage effect. Therefore, the definition of the leverage effect (rather than leverage) is the eventual difference in risk and expected rate of return resulting from equity financing and

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[7] Frank H. Knight, *Risk, Uncertainty and Profit* (Houghton Mifflin Co., 1921) p. 20.

debt financing under a diversified investment situation.[8]

Investment Indifference Curve. The general shape of the investment indifference curve is convex to the northwest (concave from the southeast) on a graph where the vertical axis measures risk and the horizontal axis measures the expected rate of return. Graphically, a move in the southeasterly direction tends to raise the level of satisfaction. This can be explained by investors' behavior. Investors prefer less risk and a higher expected rate of return; therefore, when risk is held constant investors prefer a higher expected rate of return and when the expected rate of return is held constant, investors prefer lower risk.

The convexity can be explained by risk aversion. In order to gain a given increase in the expected rate of return, greater increments of risk will be accepted at lower levels of risk. In other words, the marginal rate of substitution of the expected rate of return for risk $(\frac{\Delta\ ER}{\Delta\ RISK})$ becomes higher when the risk increases.

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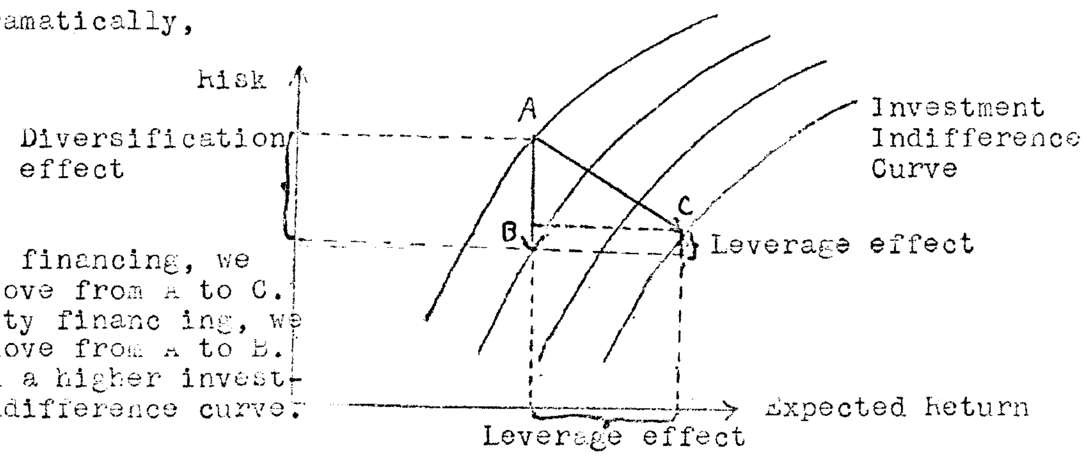
[8] Diagramatically,



By debt financing, we can move from A to C. By equity financing, we can move from A to B. C is on a higher investment indifference curve.

Assumptions

The Tax Rate. The tax factor is an institutional and a political phenomenon. It varies from country to country and from time to time. For purposes of this thesis a tax system which approximates the current U.S. system is assumed. Throughout the analysis a net income tax rate of 50 percent is used; and the interest on debt is considered deductible whereas the dividends are not.

The Interest Rate. In the first part of the argument it is assumed that the interest rate is constant at five percent, regardless of the so-called "lender's risk." Solomon comments on this point as follows:

> The trouble with this approach is that it ignores a second form of cost associated with increasing the ratio of debt to equity. This is the deterioration which increased borrowing brings about in the quality of residual net earnings, i.e., the increase in financial uncertainty. This cost is much harder to compute, but it can not be ignored.[9]

Computation Procedure

There is no mechanical problem in measuring expected rate of return. This measurement is expressed mathematically by $\frac{\Sigma Xifi}{n} / E$ where Xi is net profit after tax, fi is the probability frequency of outcome, $n = \Sigma fi$ (total frequency) and E is the amount of equity capital. More simply, the expected rate of return is the weighted average rate of return of all possible outcomes. It is also necessary to specify the measurement of risk. According to Archer and D'Ambriosio:

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[9]Ezra Solomon, op. cit., p. 80.

> The expected outcome of an investment's performance is a measure which, in general terms, indicates the center of the range of possibilities. It does not, however, tell us anything about the dispersion of the outcome from that which can be expected on the average ---. This (divergence) is what we wish to measure as risk, the extent to which an investment may turn out better or worse than expected.[10]

There are two possible measurements of dispersion. One is variance and the other is the standard deviation. The disadvantage of using the variance as a measure of dispersion is that it is not in the same units of measurements as the original data. But this disadvantage disappears when the square root of the variance is calculated, thereby expressing dispersion in terms of the standard deviation.

$$6^2 = \frac{\Sigma(x-x)^2}{N} \qquad \ldots\ldots\ldots\ldots \text{ variance}$$

$$6 = \sqrt{\frac{\Sigma(x-x)^2}{N}} \qquad \ldots\ldots\ldots\ldots \text{ standard deviation}$$

$$\frac{6}{\bar{x}} = \frac{\sqrt{\frac{\Sigma(x-x)^2}{N}}}{\bar{x}} \qquad \ldots\ldots\ldots\ldots \text{ coefficient of variation}$$

To maintain comparability between different sized investments, and to measure risk the coefficient of variation is used throughout this study.

A usual argument is that an increase in debt ratio in total financing leads to greater risk. The hypothesis of this thesis is that when equity financing is compared with debt financing, increased debt will lead to greater risk than equity financing. However, this study

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[10] Stephen H. Archer and Charles A. D'Ambriosio, Business Finance: Theory and Management (New York: The Macmillan Company, 1966) pp. 68-69.

maintains that not only is this tendency toward greater risk nullified under the diversification situation but an increase in expected rate of return also occurs. In other words, the increase in risk is in a sense a cost associated with the gain in the expected rate of return. The evaluation of debt financing should be determined by the comparison of these costs and gains.

Chapter II

THE VARIANCE RISK AND LEVERAGE

This section will discuss the relationship between the risk and leverage. As mentioned before, it is assumed that the probability distribution of the outcomes is given, and that risk rather than uncertainty is considered.

One Investment

This section will begin with the comparison of the following two simple cases.[1]

A. Equity financing. In the first example, suppose an investment of $1,000 has a fifty-fifty chance of making a 20 or 0 percent profit. The expected outcome or return (ER) would be (20% x 1/2 + 0% x 1/2) = 10%, and by following calculation its standard deviation is 10 percent.

Outcome %	Divergence from ER	$(D)^2$	Probability	$P \times (D)^2$
0.00	0.1	0.01	0.5	0.005
0.20	0.1	0.01	0.5	0.005
				0.010

Standard Deviation of Outcome (SR) = $\sqrt{0.01} = 0.1$

The coefficient of variation in terms of the rate of return is

(SR/ER) $\frac{0.10}{0.10} = 1$

[1] Stephen H. Archer and Charles A. D'Ambriosio, Business Finance: Theory and Management (New York: The Macmillan Company, 1966) pp. 77-79.

B. Equity and debt financing. In the second example, suppose a similar investment exists, except that an additional \$1,000 is borrowed at a rate of 5 percent. In this case, when the investment is successful, a profit of 35 percent (\$2,000 x 20% - \$1,000 x 5% = \$350) is earned, and when the investment fails, the loss is 5 percent (\$2,000 x 0% - \$1,000 x 5% = -\$50). The expected rate of return is (35% x 1/2 - 5% x 1/2) = 15%, and by the following calculation its standard deviation is 20 percent.

Outcome %	Divergence from ER	$(D)^2$	Probability	$P \times (D)^2$
-0.05	0.20	0.04	0.5	0.02
0.35	0.20	0.04	0.5	0.02
				0.04

The Standard Deviation of the Outcome is $\sqrt{0.04} = 0.2$

The Coefficient of Variation is $\frac{0.2}{0.15} = 1.33$

Two Investments

In the above example, the use of debt capital brings a higher expected rate of return and a higher risk than the use of equity capital, assuming the additional \$1,000 borrowed capital is added to the same investment. In other words, it is invested together with the equity capital as a unit in the same indivisible investment. This is probably unrealistic.

With large investments, the borrowed capital can be used to purchase a single investment. In this case the increased safety through diversification by borrowed capital is precluded, but profitability can be increased.

When the indivisible investment is preferred to the divisible investment, there should be a better combination of risk and profitability. In the case of the above debt financing (case B), divisible investment is presumed to be available. Therefore, there are two investments, A and B, each costing \$1,000, and the probability of success of A and B is one-half, respectively. The outcome would become:

A Probability		B Probability		Total Probability		
success	1/2	success	1/2	complete success	1/4	
success	1/2	failure	1/2	half success	1/4	1/2
failure	1/2	success	1/2	half success	1/4	
failure	1/2	failure	1/2	complete failure	1/4	

Success in both investments results in a 35 percent return (\$2,000 x 20% - \$1,000 x 5% = \$350). One success and one failure results in a 15 percent return (\$1,000 x 20% - \$1,000 x 5% = \$150). When both investments fail, 5 percent is lost (\$2,000 x 0% - \$1,000 x 5% = -\$50). Then the expected outcome is \$350 (1/4) + \$150 (1/2) - \$50 (1/4) = \$150 and the rate of return to equity capital is \$150/\$1,000 = 15%. The standard deviation is \$141.42/\$1,000 = 14.142% and the risk is 0.9428.

Number of success	Earning before interest	Interest	Net profit after interest	Probability	Divergence from ER	D^2	D^2 x p
2	\$400	50	+350	1	+200	40,000	40,000
1	200	50	+150	2	0	0	0
0	0	50	-50	1	-200	40,000	40,000

Standard Deviation of the Outcome is $\sqrt{\frac{80,000}{4}} = \sqrt{20,000} = 141.421$
(14.1421%)

Expected outcome = 15%

Risk (SR/ER) = 14.1421%/15% = 0.9428.

Comparing this result with cases A and B respectively, it can be noted that this result is not only much better than case B, but also better than case A in terms of the investment indifference curve attained. That is, by borrowing an additional \$1,000 the expected outcome was raised from 10 to 15 percent, and the standard deviation changed from 0.1 (10% of equity capital) to 0.14142 (14.142% of equity capital = \$1,000). As a result, risk decreased from 1 to 0.9428 by borrowing debt capital under the divisible situation. This situation is shown in Figure 1.

As illustrated in Figure 1, when the indivisible investment is financed by debt, both the expected rate of return and the risk increase (from A to B), but when the investment is divisible, the debt financing increases the expected rate of return and at the same time decreases risk (from A to C). Point C is absolutely superior to point A.

As a next step it is assumed that there are two investments of \$1,000 each and \$1,000 equity capital is available: the decision is whether equity or debt should be used to finance the other investment. In this case the tax effect and the depreciation effect must be considered. The income tax rate is assumed to be 50 percent and the annual depreciation cost is 5 percent of the original asset cost.

A. <u>The short run effects</u>. As illustrated in Table 1, the use of equity capital results in a one-fourth probability of getting \$200 profit (after tax) and one-half probability of getting \$100 profit (after tax) and one-fourth probability of getting \$0 profit (after tax). Then the expected rate of return is five percent (ER = \$100/\$2,000 = 5%) and

A Comparison of Divisible Investments
and Indivisible Investments.

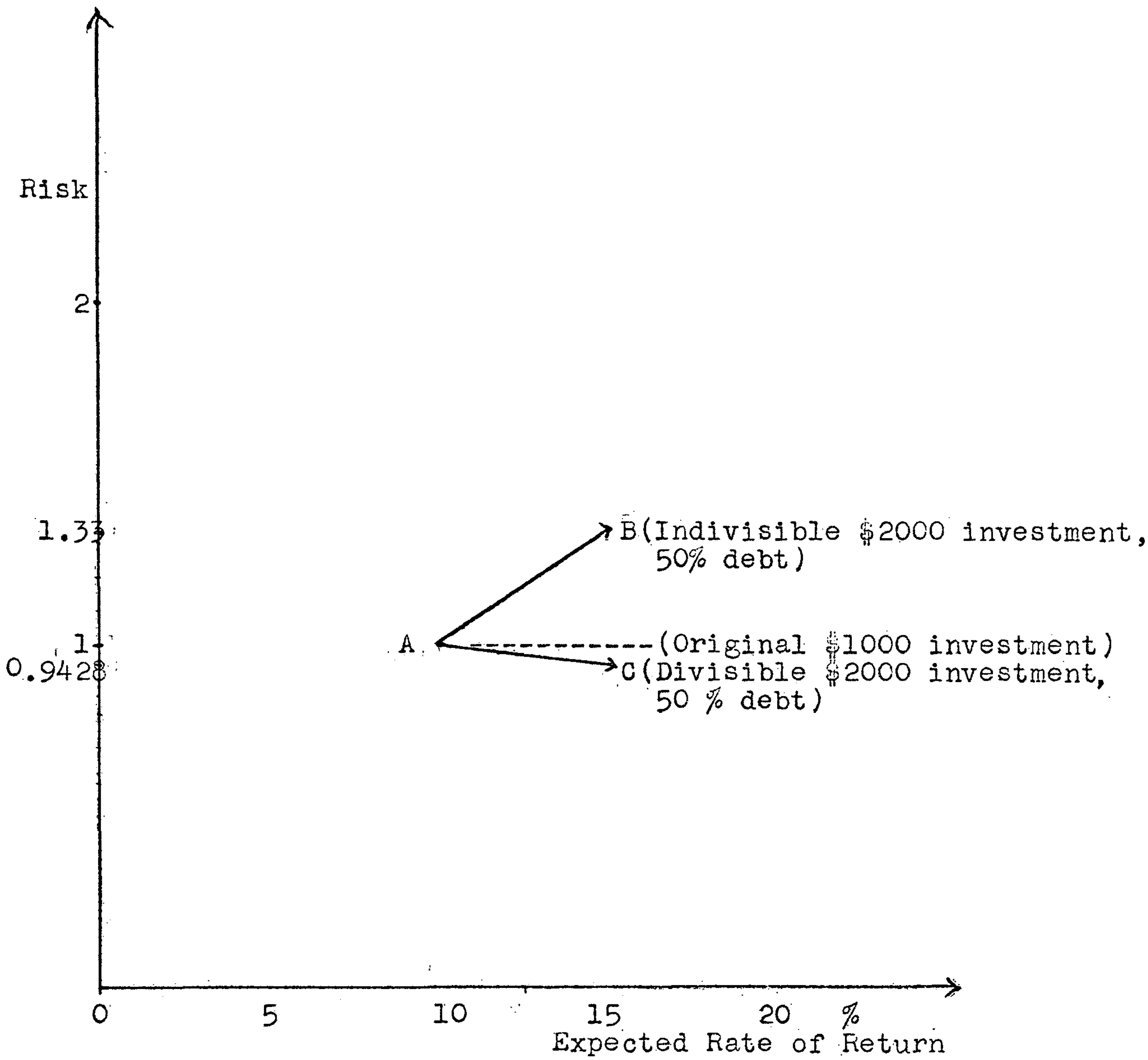

Figure 1

the coefficient of variation is 3.535 percent (SR = \$70.71/\$2,000 = 0.03535 = 3.535%) with the risk equal to $\frac{3.535}{5}$ = 0.707.

Dependence on debt capital (Table 2) necessitates paying interest whether or not a profit is made. Then the net profit after interest is, assuming successes in both investments, \$400 - \$50 = \$350; success in only one investment, \$200 - \$50 = \$150; and failure in both investments, \$0 - \$50 = -\$50.

When the profit before tax is -\$50, there is no tax, but any profit is taxed at 50 percent. The net profit after tax is respectively \$175, \$75, -\$50 (column 5 of Table 2). Using the same procedure as for equity capital, the expected return is x = 75 - $\frac{25}{4}$ = 68.75 (column 8) and the standard deviation is σ = 80.0. From the viewpoint of equity capital, the expected rate of return ER = \$68.57 = 6.875%. As a result the risk is 8.00%/6.875% = 1.163.[2]

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[2]The expected return X is obtained from Table 2 in the following way:
$\overline{X}$ = 75 (temporary mean, column 5) - $\frac{25}{4}$ (adjustment, from column 8) = 68.75.
This is eventually the same thing as following calculation (columns 5 & 7)

$$\overline{X} = \frac{\Sigma\, XiPi}{n} = \frac{175 \times 1 + 75 \times 2 - (-50) \times 1}{4}$$

$$= \frac{175 + 150 - 50}{4} = \frac{275}{4} = 68.75$$

Standard Deviation σ is obtained from column 9, Table 2.

$$\sigma = \sqrt{\frac{\Sigma(xi-\overline{x})^2}{n}} = \sqrt{\frac{25.625}{4}} = 80$$

This is expressed in terms of percentage of the investment size.

$$\sigma = \frac{\$80}{\$1,000} = 8.00\%$$

TABLE 1

MEASUREMENT OF EXPECTED RATE OF RETURN AND RISK FOR TWO INVESTMENTS (100% EQUITY)

In the short run

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax	Divergence $u=x-\bar{x}$	Probability p	U x P	U^2 x P
2	400	0	400	200	+100	1	+100	10,000
1	200	0	200	100	0	2	0	--
0	0	0	0	0	-100	1	-100	10,000
Total						4	0	20,000

TABLE 2

DO. (50% EQUITY AND 50% DEBT)

In the short run

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax	Divergence $u=x-\bar{x}$	Probability p	U x P	U^2 x P
2	400	50	350	175	100	1	100	10,000
1	200	50	150	75	0	2	0	0
0	0	50	-50	-50	-125	1	-125	+15,625
Total						4	-25	25,625

Table 1

$\bar{x} = \$100/\$2{,}000 = 5\%$

$\sigma = \sqrt{\frac{20{,}000}{4}}/\$2{,}000 = \$70.71/\$2{,}000 = 3.535\%$

Risk $= \frac{\sigma}{\bar{x}} = 3.535/5 = 0.707$

Table 2

$\bar{x} = \$68.75/\$1{,}000 = 6.875\%$

$\sigma = \sqrt{\frac{25{,}625}{4}}/\$1{,}000 = \$80.0/\$1{,}000 = 8.00\%$

Risk $= \frac{\sigma}{\bar{x}} = 8.00/6.875 = 1.163$

A comparison of Table 1 and Table 2 reveals that under the divisible situation, increasing both equity and debt capital, the expected rate of return is increased from $50 (5%) to $68.75 (6.875%). However, risk is increased from 0.707 to 1.163.

For comparative purposes the original case A (page 9, quoted table from Archer & D'Ambriosio, no tax consideration) is adjusted to Table 3 (with tax consideration), and the results are shown in Figure 2. As is illustrated in this figure, earnings stability can be increased by using diversified equity capital, but profitability cannot be improved at all. However, the use of debt financing, when the investment is divisible, increases both profitability and risk. There should be some indifference curve representing the combination of the expected rate of return and risk.[3] Generally, the shape is as follows:

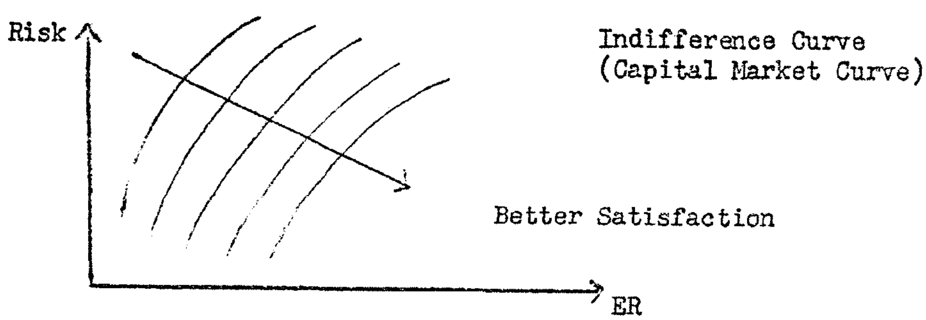

Figure 3.

[3] Investors prefer less risk and a higher expected rate of return. Therefore a higher satisfaction is obtained by shifting southeasterly to the next indifference curve. The concavity of the indifference curve can also be explained by risk aversion.

TABLE 3

MEASUREMENT OF EXPECTED RATE OF RETURN AND RISK
FOR ONE INVESTMENT (100% EQUITY)

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax	Divergence $u=x-\bar{x}$	Probability p	U x P	U^2 x P
1	200	0	200	100	+50	1	+50	2,500
0	0	0	0	0	-50	1	-50	2,500
Total						2	0	5,000

$\bar{x} = 50$ ER = 50/1000 = 5%

$\sigma = \sqrt{\frac{5000}{2}} = 50.00$ $\sigma = 50.00/1000 = 5.000\%$

$\text{Risk} = \frac{\sigma}{\text{ER}} = \frac{5.000}{5} = 1.000$

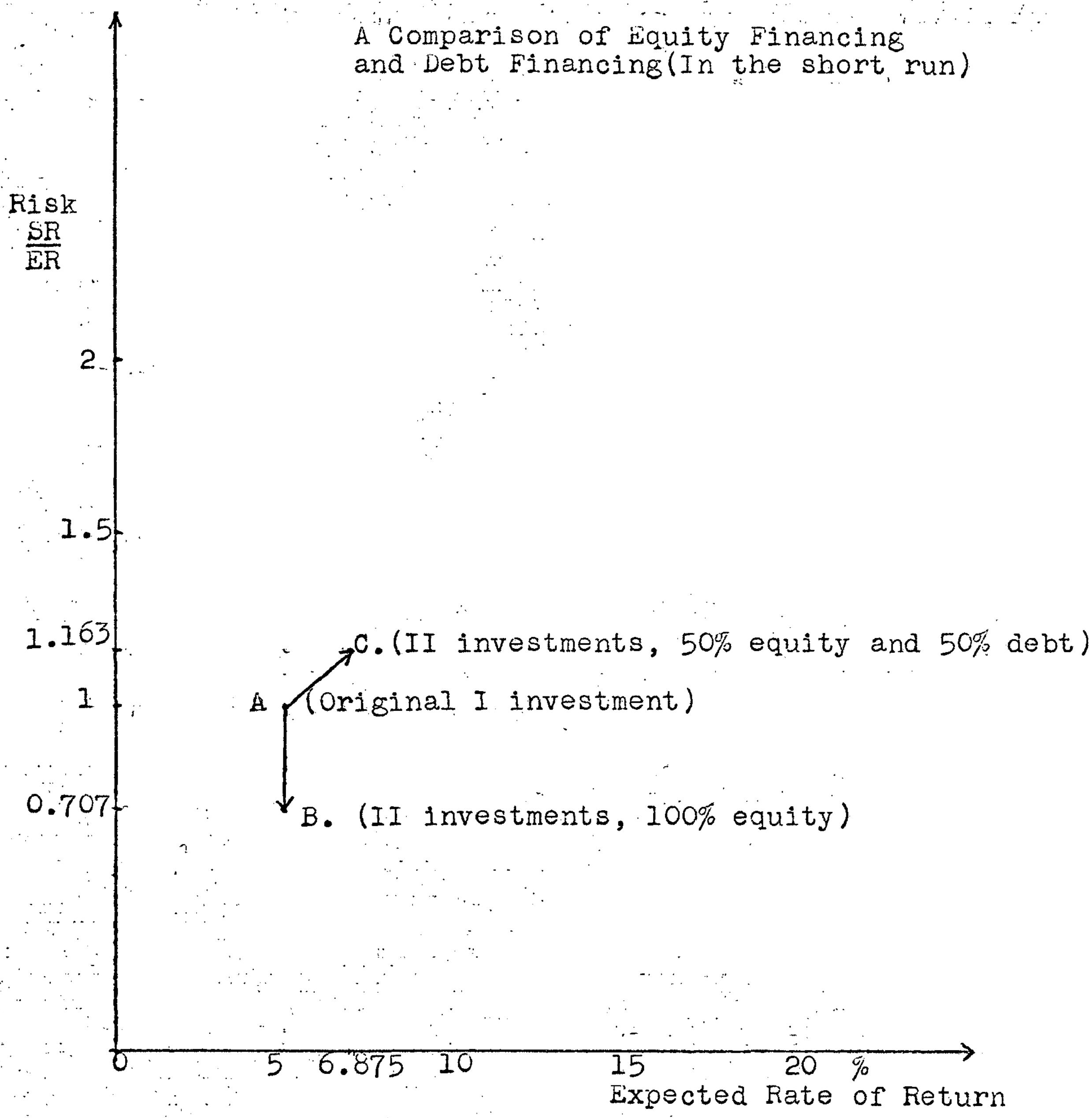
A Comparison of Equity Financing
and Debt Financing(In the short run)
Risk
SR
ER
2
1.5
1.163
1
0.707
C.(II investments, 50% equity and 50% debt)
A (Original I investment)
B. (II investments, 100% equity)
0
5
6.875
10
15
20
%
Expected Rate of Return

Figure 2

In connection with the short run risk of debt capital, it is important to consider the effect of depreciation allowances. Generally, as the number of investments increases, the total depreciation allowance increases, but the risk of running out of cash in the short run decreases conspicuously even if debt capital is used.

A hypothetical case is assumed where equity capital is not used at all. In this example annual depreciation allowances are assumed to be 5 percent of the original cost of the purchased assets and the greatest loss occurs when every investment results in complete failure; that is, earnings are zero. The amount of interest depends on the total amount of debt which is, in this case, equal to the amount of total investment.

As illustrated in Table 4, as the number of investments increase, both the amount of the maximum loss and the depreciation allowance increase at the same rate, but the probability of the greatest loss decreases conspicuously as the number of investments increase. This can be explained as follows: when there are three investments, the probability of greatest loss is $1/2^3 = 1/8$ because each investment has a 50 percent probability of failure. In the same way the probability of the maximum loss is $1/2^{10} = 1/1024$ for ten investments.

When equity capital is used, at least one unit, then depreciation less the loss is greater than zero $(B - A > 0)$, because when the equity investment fails, the loss is \$0, but the depreciation allowance is \$50.

From this discussion it can be said that even if each investment has a .5 probability of failure, the short-run risk is decreased conspicuously by borrowing more, assuming diversification. Of course,

TABLE 4

THE EFFECT OF DEPRECIATION ALLOWANCES ON SHORT-RUN RISK

Number of investment	1	2	3	4	5	6	7	8	9	10
Probability of greatest loss	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{256}$	$\frac{1}{512}$	$\frac{1}{1024}$
A Amount of greatest loss	50	100	150	200	250	300	350	400	450	500
B Depreciation	50	100	150	200	250	300	350	400	450	500
B - A	0	0	0	0	0	0	0	0	0	0

this situation is applicable to equity capital too.

B. The long run effects. So far the short-run (one year) situations have been discussed. And it was found that there is no need to worry about the short-run risk. But in the long run, even if liquidity is maintained by depleting depreciation allowances, there are serious consequences.

The major difference between the long-run and the short-run situation is that in the long run the loss in one period is not only exempted from taxes, but if the loss exceeds current profits, it can be deducted from the profit of following years. Thus, in the long run the tax is levied on the income remaining after the deduction of all previous losses: the loss strengthens the tax position.

These facts are reflected in Table 5. The expected rate of return is 7.5% ($75/$1,000), the standard deviation is 7.071% ($70.71/$1,000) and the risk is 0.9428 (7.071%/7.5%). This situation can best be explained by Figure 4. The risk and the expected outcome with 100 percent equity financing do not change between the long and short run because the use of equity never shows a loss. The same is true even if the tax effect is considered.

When both investments are compared in the long run, the situation can be improved by using debt capital with divisible investments. The point changes from C to D; that is, risk decreases from 1.163 to 0.9428 at the same time the expected outcome increases from 6.875% to 7.5%. Then the decision is between points B and D. The choice between points B and D depends chiefly on the attitude of the investor. Generally an

TABLE 5

MEASUREMENT OF EXPECTED RATE OF RETURN AND RISK FOR TWO INVESTMENTS (50% EQUITY AND 50% DEBT)

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax	Divergence $u=\frac{x-\bar{x}}{100}$	Probability p	U x P	U^2 x P
2	400	50	350	175	+1	1	1	1
1	200	50	150	75*	0	2	0	0
0	0	50	-50	-25*	-1	1	-1	1
Total						4	0	2

*The difference with Table 2 is only this part.

$\bar{x} = 75$

$\sigma = 100\sqrt{\frac{2}{4}} = 100\sqrt{\frac{1}{2}} = 100\sqrt{0.5} = 100 \times 0.7071 = 70.71$

ER = \$75/\$1000 = 7.5%

SR = \$70.71/\$1000 = 7.071%

Risk $= \frac{SR}{ER} = \frac{7.071}{7.5} = 0.9428$

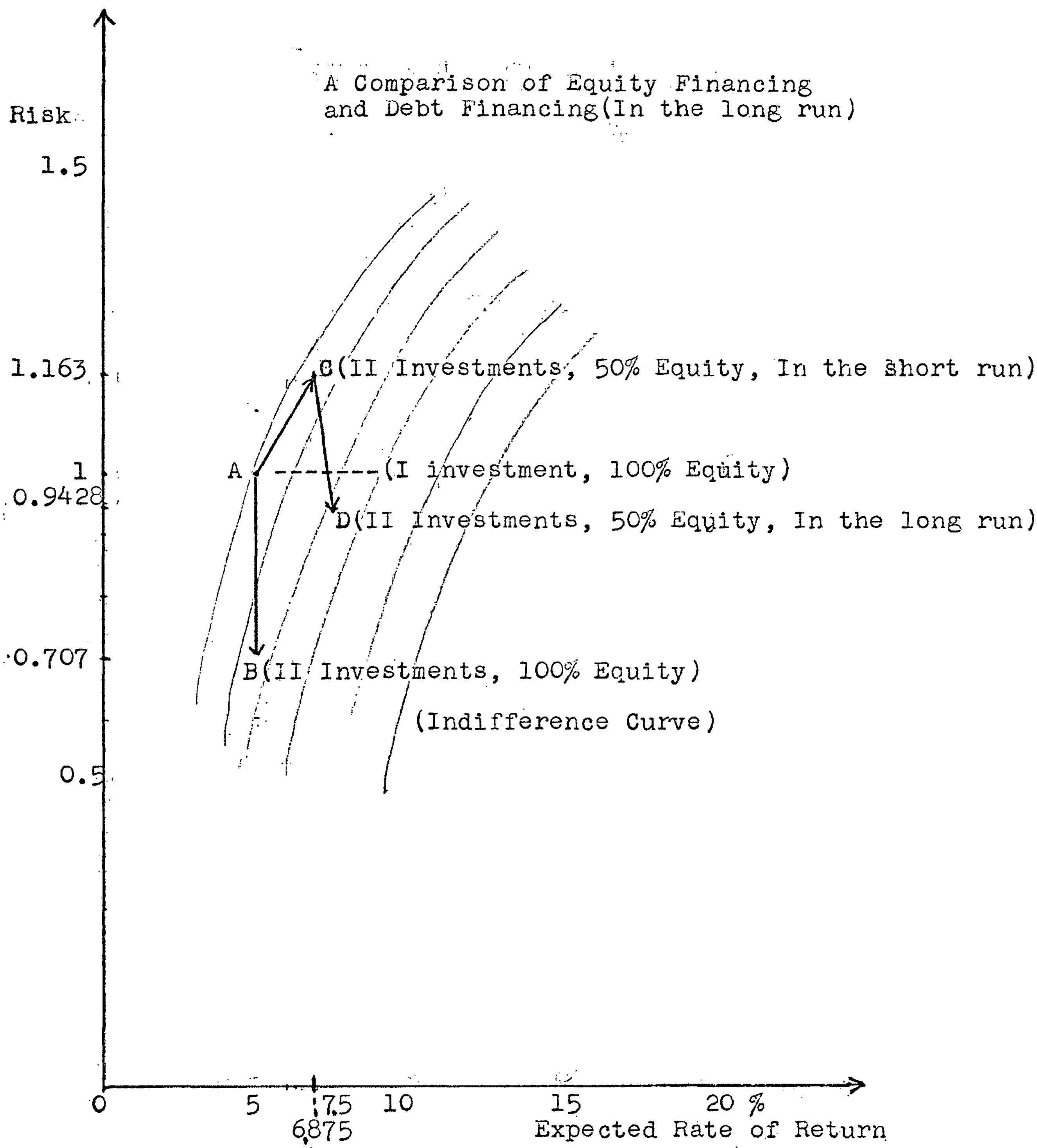
A Comparison of Equity Financing
and Debt Financing(In the long run)
Risk
1.5
1.163
1
0.9428
0.707
0.5
C(II Investments, 50% Equity, In the short run)
A
(I investment, 100% Equity)
D(II Investments, 50% Equity, In the long run)
B(II Investments, 100% Equity)
(Indifference Curve)
0
5
7.5
6.875
10
15
20 %
Expected Rate of Return

Figure 4

aggressive investor would prefer the point D to the point B. This situation can be more clearly explained by considering the following extreme case.

Graphically, the effect of diversification is vertically downward, and the effect of leverage is horizontal. Therefore, the use of debt capital in diversified situations produces both a leverage and a diversification effect (risk decreasing effect), whereas the use of equity financing in diversified situations does not improve earnings, but only decreases risk.[4]

The comparison between debt and equity financing in terms of investment indifference curves reveals that the use of more debt changes the point from northwest to southeast; that is, it moves along the shortest distance to the higher indifference curve, whereas the use of debt financing decreases risk but does not increase profitability; therefore, the direction of the move is not the shortest cut to a higher indifference curve.

Extreme Cases:

A. Equity financing. Assume an investment in ten different projects, each of which costs $1,000 and has a fifty-fifty probability

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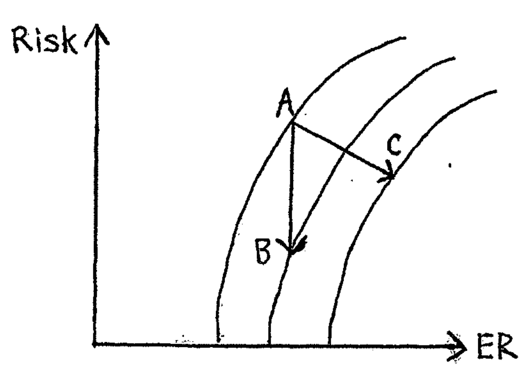

Diagramatically, when the distance AB equal to AC, the higher investment indifference curve can be reached by AC rather than AB. This direction A - C is the shortest cut to higher investment indifference curve.

of returning 20 percent or 0 percent. Each investment is statistically independent. The probability of ten successes out of ten projects will be $10^{C}10(1/2)^{10}\ (1/2)^{0} = 1/1024$, the probability of nine successes out of ten will be $10^{C}9(1/2)^{9}\ (1/2)^{1} = 10/1024$ and so on, as shown in Table 6.

This table indicates that average expected rate of return after taxes is 5 percent (\$500/\$10,000), the standard deviation is 1.58 percent (\$1.58/\$10,000) and the risk is 0.32 (1.58/5).

B. Debt financing. In this case, assume that the first unit of investment requires equity capital and that all additional sources of money are obtained only through debt financing. Furthermore, assume that the rate of interest is 5 percent. It can be seen from Table 7 that the expected rate of return is 27.5 percent (\$275/\$1,000), the standard deviation is 15.8 percent (\$158/\$1,000), and the risk is 0.57 percent (15.8/27.5). The probability that the profit is less than zero (a loss) is 1.74 (Z = \$27.5 - \$0/15.8 = 1.74 standard deviation) and from the table of a normal curve area, it is 0.4591. The probability of incurring a loss is 4.09 percent (0.5 - 0.4591).

A comparison of the two examples is illustrated in Figure 5. Point M on the higher investment indifference curve is preferable to point K, showing the investors' preference to debt financing. The use of equity capital does not improve the expected rate of return; it only decreases risk. Debt capital, however, increases the expected rate of return, also decreasing the risk.

TABLE 6

MEASUREMENT OF EXPECTED RATE OF RETURN AND RISK FOR TEN INVESTMENTS (100% EQUITY)

In the long run
In the short run

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax(x)	Proba-bility p	Diver-gence $U=\frac{x-\bar{x}}{100}$	P x U	P x U^2
10	2,000	0	2,000	1,000	1	5	5	25
9	1,800	0	1,800	900	10	4	40	160
8	1,600	0	1,600	800	45	3	135	405
7	1,400	0	1,400	700	120	2	240	480
6	1,200	0	1,200	600	210	1	210	210
5	1,000	0	1,000	500	252	0	0	0
4	800	0	800	400	210	-1	-210	210
3	600	0	600	300	120	-2	-240	480
2	400	0	400	200	45	-3	-135	405
1	200	0	200	100	10	-4	-40	160
0	0	0	0	0	1	-5	-5	25
Total					1,024	0	0	2,560

$\bar{x} = 500$

$\delta = 100\sqrt{\frac{2560}{1024}} = 100 \times 1.58 = 158$

The Expected Rate of Return = \$500/\$10,000 = 5%

Standard Deviation = \$158/\$10,000 = 1.58%

Risk = 1.58/5 = 0.32

TABLE 7

MEASUREMENT OF EXPECTED RATE OF RETURN AND RISK FOR TEN INVESTMENTS (10% EQUITY, 90% DEBT)

In the long run

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax (x)	Divergence $U = \frac{x - \bar{x}}{100}$	Probability p	U x P	U^2 x P
10	2,000	450	1,550	775	5	1	5	25
9	1,800	450	1,350	675	4	10	40	100
8	1,600	450	1,150	575	3	45	135	405
7	1,400	450	950	475	2	120	240	480
6	1,200	450	750	375	1	210	210	210
5	1,000	450	550	275	0	252	0	0
4	800	450	350	175	-1	210	-210	210
3	600	450	150	75	-2	120	-240	480
2	400	450	- 50	- 25	-3	45	-135	405
1	200	450	- 250	-125	-4	10	- 40	160
0	0	450	- 450	-225	-5	1	- 5	25
Total					0	1,024	0	2,560

$\bar{x} = 275$

$$\sigma = 100\sqrt{\frac{2560}{1024}} = 100 \times \sqrt{2.5} = 100 \times 1.58 = 158$$

The Expected Rate of Return = \$275/\$1000 = 27.5%

Standard Deviation of Outcome = \$158/\$1000 = 15.8%

Risk = 15.8%/27.5% = 0.57

A Comparison of Equity Financing
and Debt Financing(Ten Investments)

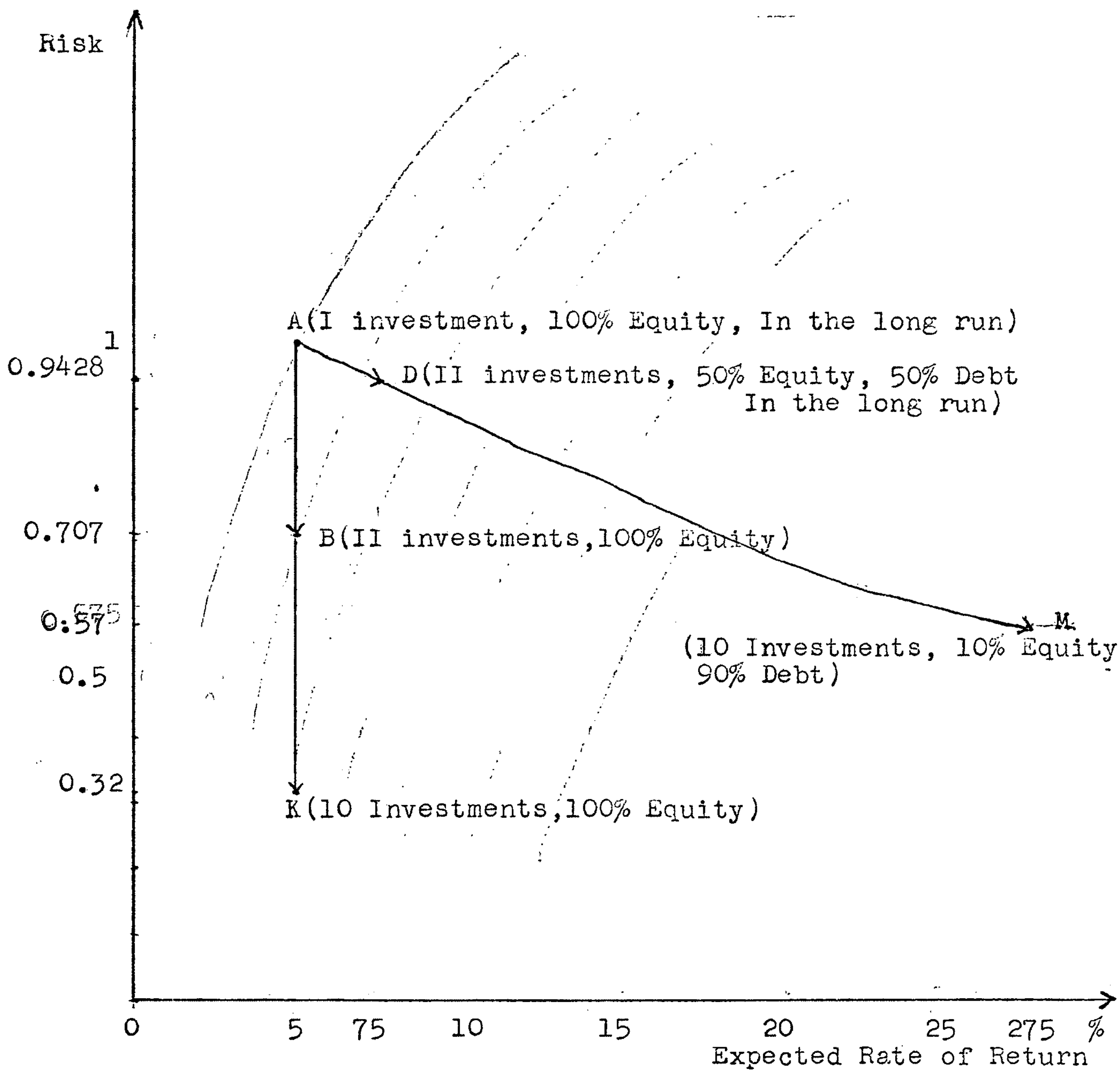

Figure 5

Lender's Risk

A generally accepted principle in finance is that as the percentage of the debt in total financing becomes larger the lender's risk also increases. As a result, lenders request higher rates of interest to compensate for the increased risk and accordingly, the cost of capital increases.[5] But insofar as the above examples are concerned, the lender's risk decreases as the percentage of the debt in total financing increases.

By definition, lender's risk is the risk that the lenders will not receive either principal or interest or both. There is no lender's risk if no debt is assumed. Lender's risk appears only when at least one unit of debt capital is used. Table 5 reveals that the probability that the lenders will not be repaid their principal is one-fourth, and the total amount of their loss is \$500 (\$25 x 20 years). The expected loss is calculated in the same way as the expected rate of return on page 11; that is \$125 (1/4 x \$500).

The following expected loss can be computed from Table 7 when one unit of equity and nine units of debt are used:

---

[5]Ezra Solomon, "Leverage on the Cost of Capital." Foundations for Financial Management. A Book of Readings. by James Van Horne. The quotation is from p. 409.

> "But in practice, ki (cost of debt capital), the average rate of interest paid on debt, must rise as leverage is increased. For extreme leverage positions, i.e., as the company approaches an all-debt situation, it is clear that ki will be at least equal to ko (cost of overall capital). Given the general attitude of bondholders and bond rating agencies, it is highly likely that ki will be above ko for positions of extreme leverage."

Total loss	Probability	Expected loss
\$25 x 20 years = \$500	45/1,024	22,500/1,024
\$125 x 20 years = \$2,500	10/1,024	25,000/1,024
\$225 x 20 years = \$4,500	1/1,024	4,500/1,024
		52,000/1,024

\$52,000/1,024 = \$50.

The comparison of these two examples indicates that the expected loss decreases conspicuously from \$125 to \$50 with an increase in debt from 1 unit to 9 units. But both cases do not cause any loss as far as the \$1,000 equity capital is concerned. This is the entrepreneur's and not the lender's risk. The lender's risk is only the deficit less the equity capital; in both cases the interest is already paid. It can therefore be concluded from the above discussion that lender's risk decreases with more debt financing. Theoretically, the corporation can require lenders to cut down the rate of interest because of the lender's increased safety.

The Effect of a Change in Variance

In the above example it was assumed that success resulted in a 20 percent return and failure resulted in a 0 percent return. Although the average of the rate of return is 10 percent, the total standard deviation doubles when the standard deviation of each investment is doubled. Therefore, the risk is doubled. For example, suppose an investment has a fifty-fifty chance of yielding a 40 percent profit or a 20 percent loss. The expected rate of return is 10 percent, but the standard deviation and the risk become three times what they were in the original example.

Depending on the nature of the industry, different standard deviations can be applied; but the validity of this approach in comparing a company using higher proportions of debt and a company using higher proportions of equity does not change, because both companies are assumed to be identical except in their capital structures. There is no change in the standard deviation because of a difference in the capital structures. This situation is illustrated in Tables 8 and 9. As the proportion of equity capital increases, the relative standard deviation applicable to the equity capital decreases.

The Effect of a Change in the Rate of Expected Return

As Figure 6 and Table 10 illustrate, the risk situation changes as the expected rate of return (before taxes and interest) goes down. With a 10 percent expected rate of return, the effect of diversification is much larger than the leverage effect. In the case of one investment, the risk is 1.000; but by borrowing one unit more under diversification the risk decreases to 0.707. If an additional nine units of debt capital are added, the risk decreases to 0.575.

When the expected rate of return goes below 8.5 percent, the entrepreneur's risk of using debt capital exceeds the risk of the original one unit equity investment. With 90 percent debt financing and 10 percent equity financing, the entrepreneur's risk reaches infinity as the expected rate of return reaches 4.5 percent (5% x 90%); any rate below 4.5 percent makes the lender's risk a reality. Similarly, one unit of equity capital and one unit of debt capital increase the entrepreneur's risk infinitely as the expected rate of return

TABLE 8

MEASUREMENT OF EXPECTED RATE OF RETURN AND RISK FOR ONE INVESTMENT (100% EQUITY)

Number of success	Earning	Interest	Net profit after tax	Divergence $U = x - \bar{x}$	Probability p	U x P	U^2 x P
1	400	0	200	+150	1	+150	22,500
0	-200	0	-100	-150	1	-150	22,500
Total				0	2		45,000

The Expected Rate of Return ER = \$50/\$1000 = 5%

The Standard Deviation of the Outcome $\delta = \sqrt{\frac{45,000}{2}}$ /\$1000 = \$150.0/\$1000 = 15.00%

Risk = δ/ER = 15.00/5 = 3.000

This risk is three times that of Table 3.

TABLE 9

MEASUREMENT OF THE EXPECTED RATE OF RETURN AND RISK
FOR ONE INVESTMENT (100% DEBT)

Number of success	Earning	Interest	Net profit after tax	Divergence $U=x-\bar{x}$	Probability p	U x P	U^2 x P
1	400	50	175	150	1	+150	22,500
0	-200	50	-125	-150	1	-150	22,500
Total				0	2		45,000

The Expected Rate of Return ER = \$25/\$1000 = 25%

The Standard Deviation of the Outcome $\sigma = \sqrt{\frac{45,000}{2}} / 0 = +\infty$

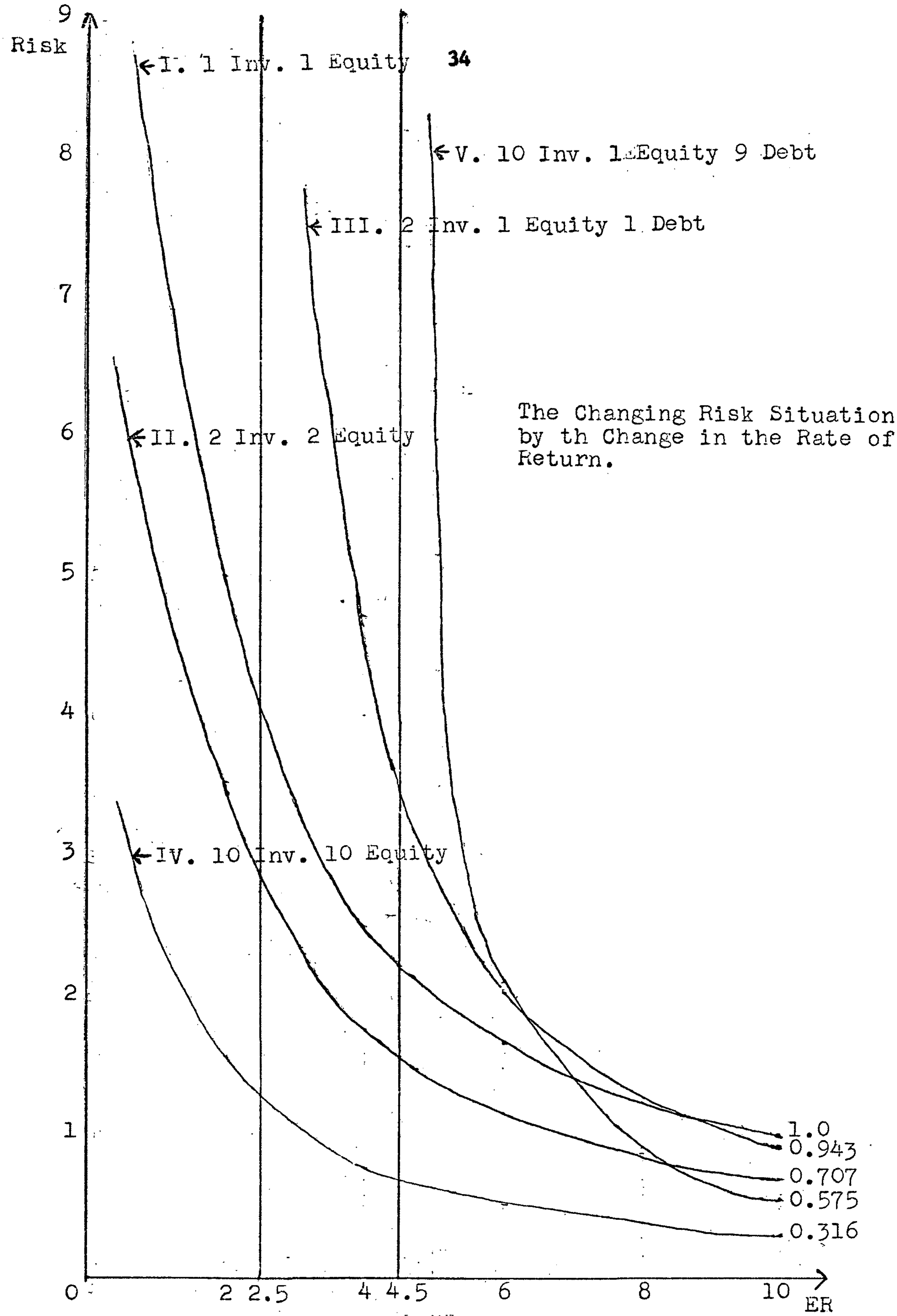
9
Risk
8
7
6
5
4
3
2
1
0
2
2.5
4
4.5
6
8
10
ER
I. 1 Inv. 1 Equity
II. 2 Inv. 2 Equity
III. 2 Inv. 1 Equity 1 Debt
IV. 10 Inv. 10 Equity
V. 10 Inv. 1 Equity 9 Debt
The Changing Risk Situation by th Change in the Rate of Return.
1.0
0.943
0.707
0.575
0.316

Figure 6

TABLE 10

THE CHANGING RISK SITUATION BY THE CHANGE IN THE RATE OF RETURN

Complete Success	20	18	16	14	12	10
Average Earning Rate	10	8	6	4	2	0
Complete Failure	0	-2	-4	-6	-8	-10
I. One Investment - 100% Equity						
After Tax Profit (P)	5	4	3	2	1	0
Standard Deviation(σ)	5.0	5.0	5.0	5.0	5.0	5.0
Risk = σ/P	1.0	1.25	1.666	2.5	5.0	0.0
II. Two Investments - 100% Equity						
After Tax Profit (P)	5	4	3	2	1	0
Standard Deviation(σ)	3.535	3.535	3.535	3.535	3.535	3.535
Risk = σ/P	0.707	0.884	1.178	1.767	3.535	0.0
III. Two Investments - 50% Equity, 50% Debt						
After Tax Profit (P)	7.5	5.5	3.5	1.5	-0.5	-2.5
Standard Deviation(σ)	7.071	7.071	7.071	7.071	7.071	7.071
Risk = σ/P	0.943	1.285	2.020	4.714	(-14.142)	(-2.828)
IV. Ten Investments - 100% Equity						
After Tax Profit (P)	5	4	3	2	1	0
Standard Deviation(σ)	1.58	1.58	1.58	1.58	1.58	1.58
Risk = σ/P	0.316	0.395	0.527	0.79	1.58	0.0
V. Ten Investments - 10% Equity, 90% Debt						
After Tax Profit (P)	27.5	17.5	7.5	-2.5	-12.5	-22.5
Standard Deviation(σ)	15.8	15.8	15.8	15.8	15.8	15.8
Risk = σ/P	0.575	0.903	2.107	(-6.32)	(-1.274)	(-0.702)

reaches 2.5 percent; when the rate goes below 2.5 percent the lender's risk is apparent.

In the case where debt capital is not used at all, the lender's risk does not exist, but the entrepreneur's risk increases infinitely as the expected rate of return approaches 0. Therefore, the potential lender's risk is apparent when the expected rate of return drops below 0 percent. In any case, absolute security can not be obtained in the world of business where risk is always inherent.

Chapter III

CONSTANT AND CHANGING INTEREST RATES

Re-examination of the Constant Interest Rate

In the previous discussion, the interest rate was assumed to be a constant 5 percent regardless of the ratios of debt to equity, which may be unrealistic. Therefore, it would be useful to examine the consequences when the interest is assumed to vary. In this case the first lender is assumed to have prior right to a specified amount of principal in the event of liquidation. In the same way the second lender has priority over the third lender and so forth. Therefore, the last lender's risk is greater than the second-to-last lender's risk and always the nth lender's risk is greater than the n-1th lender's risk.

As a matter of fact, as the lender's risk becomes larger, the interest rate which they charge becomes higher. But the important fact is that the risk of the additional n+1th lender is greater than the risk of the current nth lender. If a constant ratio between the lender's risk and the interest rate is assumed, then the interest rate is doubled when the lender's risk is doubled.

However, the measurement of risk is one of the most difficult problems. The lender's risk was previously defined as the probability that the lender will not be repaid either principal or interest or both. For simplicity of calculation, the same probability distribution is assumed every year. Therefore, when the mean earnings during the first year are $68.75, the total earnings for 20 years are $1,375

(\$68.75 x 20 years), and when the standard deviation of the first year is \$80.00, the total standard deviation for 20 years is \$1,600 (\$80.00 x 20 years). If the earnings are retained at the end of the 20th year the total equity capital should be \$2,375 (original equity \$1,000 + retained earnings \$1,375). Then the probability that the total value of equity becomes less than zero can be obtained by the following computation:

$$Z = \frac{2375 - 0}{1600} = 1.4843 \text{ (standard deviation)}$$

(from normal curve area table) 0.4306 --- 43.06%

0.5 - 0.4306 = 0.0694 --- 6.94%

Therefore, the risk of the first lender is 6.94 percent. This situation is illustrated in Case 1.

As long as a dividend policy is maintained where dividends are paid only after the payment of interest, and only when a profit is realized, then a change in dividend policy does not effect the lender's risk. The corporation will not pay a dividend when net profits after tax are less than zero. Therefore, the absolute amount of standard deviation decreases annually as long as they pay yearly dividends.

As illustrated in Case 2, the use of one more unit of debt increases the risk of the lender from 6.94 percent to 7.21 percent, and the risk of the first lender decreases from 6.94 percent to 2.62 percent. But the important comparison is the risk of the first lender in Case 1 and the risk of the second lender in Case 2. If the risk of the latter is larger than the risk of the former, the second lender will charge a higher interest rate. In this case the risk of the second lender in

Case 2 (7.21%) is more than the risk of the first lender in Case 1 (6.94%). Therefore, the second lender will charge more than the first lender--that is, 5.2 percent. However, the risk of the last lender in Case 3 is 6.40 percent and this is smaller than both the risks of the last lender in Case 2 (7.21%) and the last lender in Case 1 (6.94%). Also, when junior debt is acquired, the risk of the senior lender decreases rapidly.

Two units of debt make the first lender's risk 2.62 percent and three units of debt decrease the first lender's risk to 0.73 percent. The corporation can demand a lower interest rate from the first lender, because his security is increased by the increase of the junior debt. But there are certain disadvantages underlying this theory.

First, economic fluctuation is assumed to be negligible; in other words business risk is disregarded and the discussion is limited to financial risk. However, viewing the absence of any serious economic depression since 1930 and the general upward trend of the economy, the above example can be interpreted more favorably for debt financing. Secondly, the average earning rate is assumed to be 10 percent, but this rate may be decreased as the number of investments increases. Lastly, the model presented is only applicable to economies where free competition exists.

Changing Interest Rate

In the previous section the assumption that interest rates are constant regardless of the ratio of debt to equity was examined. This section will analyze and build a model which would include increasing interest rates. One of the basic problems is to attach a numerical value to these changing interest rates. It is certain that when the lender's risk increases, interest rates tend to increase. But the lender's risk can not be considered independently of business risk since it is not usually forecasted with any numerical precision. For this reason interest rates of 5, 5.25, and 5.5 percent...7.00% are arbitrarily assigned to the first, second, third,...ninth lender respectively. The situation is then analyzed in Cases 6, 7 and 8. Table 11 provides a comparison of the first and second sections of this chapter.

Table 11 shows that when the lender determines the interest rate according to his risk (in this case the lenders are assumed to demand at least a 5 percent interest rate regardless of the risk), each additional lender's risk except the second lender's risk goes down. The last lender's risk (9th lender) is as low as 2.75 percent. In the same way, when increasing interest rates are assumed, the additional lender's risk goes down but the decreasing rate is not as conspicuous as it was in the former case. This indicates that the lender's risk will increase if the interest rate increases rapidly, which in turn, suggests that the increasing rate of interest is not the result, but the cause, of the increasing risk of lenders.

CASE 1

Equity:1
Debt:1

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax (x)	Divergence $U=x-\bar{x}$	Probability p	UxP	U^2xP
2	400	50	350	175	100	1	100	10.000
1	200	50	150	75	0	2	0	0
0	0	50	-50	-50	-125	1	-125	15.625
Total					-25	4	-25	25.625

$\bar{x}$ = \$68.75 $\qquad \bar{x}$ = 68.75 x 20 = 137.5

σ = \$80.00 $\qquad \sigma$ = 80.00 x 20 = 1600

$\bar{x} + E$ = 1375 + 1,000 = 2,375

$$z = \frac{2,375 - 0}{1,000} = 1.48 \text{ - - - - - } 0.4306 \text{ - - - - - } 43.06\%$$

The risk of the first lender - - - - - 50% - 43.06% = 6.94%

CASE 2

Equity:1
Debt:2

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax (x)	Divergence $U=\frac{x-\bar{x}}{50}$	Probability p	UxP	U^2xP
3	600	100	500	250 150	3	1	3	9
2	400	100	300	150 50	1	3	3	3
1	200	100	100	50 -50	-1	3	-3	3
0 0	0	100	-100	-700 -200	-4	1	-4	16
Total					-1	8	-1	31

$\bar{x} = 100 - 1/8 \times 50 = 100 - 6.25 = 93.75$

$\sigma = 50\sqrt{31/8} = 50\sqrt{3.875} = 50 \times 1.97 = 98.5$

$\bar{x} = 93.75 \times 20 = 1875$

$\sigma = 98.5 \times 20 = 1970$

$\bar{x} + E = 1,875 + 1,000 = 2,875$

$Z = \frac{2,875 - 0}{1,970} = 1.46$ - - - - - 0.4279 - - - - - 42.79%

The risk of the second lender - - - - - 50% - 42.79% = 7.21%

The risk of the first lender - - - - - $Z = \frac{2,875 + 1,000}{1,970} = 1.94$ - - - - - 0.4738

50% - 47.38% = 2.62%

CASE 3

Equity:1
Debt:3

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax (x)	Divergence $U=\frac{x-\bar{x}}{25}$	Probability p	UxP	U^2xP
4	800	152	648	324	8	1	8	64
3	600	152	448	224	4	4	16	64
2	400	152	248	124	0	6	0	0
1	200	152	48	24	-4	4	-16	64
0	0	152	-152	-152	-11	1	-11	121
Total					-3	16	-3	313

$\bar{x} = 124 - 3/16 \times 25 = 124 - 4.68 = 119.32$

$\sigma = 25\sqrt{313/16} = 25\sqrt{19.56} = 25 \times 4.42 = 110.50$

$\bar{x} = 119.32 \times 20 = 2{,}386$

$\sigma = 110.50 \times 20 = 2{,}210$

$\bar{x} + E = 2{,}386 + 1{,}000 = 3{,}386$

$Z = \frac{3.386}{2.210} = 1.53$ - - - - - 0.4370 - - - - - 43.7%

The risk of the last lender - - - - - 50% - 43.7% = 6.4%

The risk of the first lender - - - - - $Z = \frac{3.386 \quad 2.000}{2.210} = \frac{5.386}{2.210} = 2.44$ - - - - 49.27%

50% - 49.27% = 0.73%

<u>CASE 4</u>

Equity:1
Debt:4

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax (x)	Divergence $U = \frac{x-\bar{x}}{50}$	Probability p	UxP	U^2xP
5	1000	202	798	399	5	5	5	25
4	800	202	598	299	3	5	15	45
3	600	202	398	199	1	10	10	10
2	400	202	198	99	-1	10	-10	10
1	200	202	-2	-2	-3	5	-15	45
0	0	202	-202	-202	-7	1	-7	49
Total					-2	32	-2	184

$\bar{x} = 149 - 2/32 \times 50 = 149 - 3.12 = 145.88$

$\sigma = 50\sqrt{184/32} = 50\sqrt{5.75} = 50 \times 2.40 = 120$

$\bar{x} = 145.88 \times 20 = 2917.6$

$\sigma = 120 \times 20 = 2400$

$\bar{x} + E = 2{,}918 + 1{,}000 = 3{,}918$

$z = \frac{3{,}918 - 0}{2{,}400} = 1.63$ - - - - - 0.4484 - - - - - 44.84%

The risk of the last lender - - - - - 50% - 44.84% = 5.16%

CASE 5

Equity:1
Debt:9

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax (x)	Diver-gence $U = \frac{x-\bar{x}}{100}$	Proba-bility p	UxP	U^2xP
10	2000	450	1550	775	5	1	5	25
9	1800	450	1350	675	4	10	40	100
8	1600	450	1150	575	3	45	135	405
7	1400	450	950	475	2	120	240	480
6	1200	450	750	375	1	210	210	210
5	1000	450	500	275	0	252	0	20
4	800	450	350	175	-1	210	-210	210
3	600	450	50	75	-2	120	-240	480
2	400	450	-50	-50	-3.25	45	-146.25	475.35
1	200	450	-250	-250	-5.25	10	-52.5	275.62
0	0	450	-450	-450	-7.25	1	-7.25	52.56
Total					-3.75	1024	-26	2773.53

$\bar{x} = 275 - \frac{26}{1024} \times 100 = 275 - 2.54 = 272.46$

$\sigma = 100\sqrt{\frac{2774}{1024}} = 100\sqrt{2.709} = 100 \times 1.646 = 164.6$

$\bar{X} = 272.46 \times 20 = 5,449.2$

$\sigma = 164.6 \times 20 = 3292.0$

$\bar{X} + E = 1,000 + 5,449.2 = 6,449.2$

$Z = \frac{6,449 - 0}{3,292} = 1.96$ - - - - - 0.4750 - - - - - 47.50%

The risk of the last lender - - - - - 50% - 47.50% = 2.75%

CASE 6

Equity:1
Debt:2

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax (x)	Divergence $U=x-\bar{x}$	Probability p	UxP	U^2xP
3	600	102.5	497.5	248.75	156.41	1	156.41	24,464
2	400	102.5	297.5	148.75	56.41	3	169.23	9,546
1	200	102.5	97.5	48.75	-43.59	3	-130.77	5,700
0	0	102.5	-102.5	-102.5	-194.84	1	-194.84	37,962
Total					[illegible]	8	[illegible]	77,672

$\bar{x} = \frac{738.75}{8} = 92.34$

$\sigma = \sqrt{\frac{77,672}{8}} = \sqrt{9,709} = 98.5$

$\bar{x} = 92.34 \times 20 = 1846.8$

$\sigma = 98.5 \times 20 = 1970$

$\bar{x} + E = 1,847 + 1,000 = 2,847$

$z = \frac{2,847 - 0}{1,970} = 1.440$ - - - - - 0.4251 - - - - - 42.51%

The risk of the second lender of - - - - 50% - 42.51% = 7.49%

CASE 7

Equity:1
Debt:3

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax (x)	Divergence $U = \frac{x - \overline{x}}{100}$	Probability p	UxP	U^2xP
4	800	157.5	642.5	321.25	2	1	2	4
3	600	157.5	442.5	221.25	1	4	4	4
2	400	157.5	242.5	121.25	0	6	0	0
1	200	157.5	42.5	21.25	-1	4	-4	4
0	0	157.5	-157.5	-157.5	-2.78	1	-2.78	7.7
Total					-0.78	16	-0.78	19.7

$\overline{x} = 121.25 - \frac{0.78}{16}(100) = 121.25 - 4.87 = 116.38$

$\sigma = 100\sqrt{\frac{19.7}{16}} = 100\sqrt{1.23} = 100 \times 1.1 = 110$

$\overline{x} = 116.38 \times 20 = 2327.6$

$\sigma = 110 \times 20 = 2200$

$\overline{x} + E = 3,328$

$Z = \frac{3,328}{2,200} = 1.51$ - - - - - .4345 - - - - - 43.45%

The risk of the third lender - - - - - 50% - 43.45% = 6.55%

CASE 8

Equity:1
Debt:9

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax (x)	Divergence $U = \frac{x-\bar{x}}{100}$	Probability p	UxP	U^2xP
10	2000	540	1460	730	5	1	5	25
9	1800	540	1260	630	4	10	40	160
8	1600	540	1060	530	3	45	135	405
7	1400	540	860	430	2	120	240	480
6	1200	540	660	330	1	210	210	210
5	1000	540	460	230	0	252	0	0
4	800	540	260	130	-1	210	-210	210
3	600	540	60	30	-2	120	-240	480
2	400	540	-140	-140	-3.7	45	-166.5	616.05
1	200	540	-340	-340	-5.7	10	-57	324.9
0	0	540	-540	-540	-7.7	1	-7.7	59.29
Total					-5.1	1024	-51.2	2970.24

$$\bar{x} = 230 - \frac{51.2}{1024}(100) = 230 - 5 = 225$$

$$\sigma = 100\sqrt{\frac{2970 - (51/1024)^2}{1024}} = 100\sqrt{\frac{2970}{1024}} =$$

$$100\sqrt{29} = 100 \times 1.7 = 170$$

$$\bar{x} = 225 \times 20 = 4500$$

$$\sigma = 170 \times 20 = 3400$$

$$\bar{x} + E = 4{,}500 + 1{,}000 = 5{,}500$$

$$z = \frac{5{,}500 - 0}{3{,}400} = 1.617 \text{ - - - - - } 0.4463 \text{ - - - - - } 44.63\%$$

The risk of the last lender - - - - -50% - 44.63% = 5.37%

TABLE 11

THE CHANGING RISK SITUATION BY THE CHANGE IN THE RATE OF INTEREST

Number of Investments	Number of Debts	Section I Situation		Section II Situation (arbitrary changing interest situation)	
		Probability of default	Interest rate	Interest rate	Probability of default
1	0	--	--	--	--
2	1	6.94	5.0	5.0	6.94
3	2	7.21	5.2	5.25	6.94
4	3	6.40	5.0	5.50	6.55
5	4	5.16	5.0	5.75	--
6	5	--	--	6.00	--
7	6	--	--	6.25	--
8	7	--	--	6.50	--
9	8	--	--	6.75	--
10	9	2.75	5.0	7.00	5.37

Section I Situation: The interest rate is determined by the risk rate.

Section II Situation: The interest rate is determined arbitrarily.

Chapter IV

SUMMARY

In general, a model was used in which additional financing was achieved alternatively by debt and equity. This is in contrast to the conventional model which assumes that the total amount of capital is fixed, with only the debt-equity ratio changing. Using the new model an attempt was made to answer the pertinent question: how should the additional investments be financed? The solutions are summarized as follows: (1) under diversification, risk is decreased significantly by increasing total capital by either equity or debt financing, i.e., the diversification effect is always positive; (2) profitability cannot be increased by diversification--the diversification effect is limited to decreasing risk; (3) when total capital is increased by equity financing, the expected rate of return is not improved, this change serves only to decrease risk; (4) when the total capital is increased by debt financing the expected rate of return is improved and the risk decreases; (5) when debt and equity financing are compared through diversification, debt financing enables one to reach a higher investment indifference curve (higher level of satisfaction as determined by the combination of risk and expected rate of return). In other words, debt financing activates a leverage effect <u>and</u> a diversification effect, while equity financing yields only a diversification effect; (6) by increasing the total fixed capital, the short-term risk is decreased significantly by increased depreciation allowances; and (7) in certain diversified investment situations the lender's risk goes down as the total amount of debt capital increases. In this case there are no special reasons which justify imposing higher capital charges as the debt proportion in total financing increases. In other words, the assumed constant rate of interest appears to be a rather reasonable assumption within the framework of this model.

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LEVERAGE AND RISKY INVESTMENTS

by

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Chapter I

INTRODUCTION

Purpose of the Study

The purpose of this thesis is to construct a model that will demonstrate that a higher investment indifference curve can be reached (higher level of satisfaction) in terms of a combination of risk and expected rate of return when more debt is used, assuming diversification.

The Problem

The methods of corporate financing are traditionally classified into two major groups: one is equity financing and the other is debt financing. The rationale for this classification is that the nature of funds are different in two main respects. First, the interest payment on debt is fixed by contract, whereas dividend payments represent a variable cost which can vary freely in accordance with the performance of the business or enterprise. Second, interest payments on debt are deductible for corporate income tax purposes, whereas dividends are not.[1]

According to Solomon there are three problems to which financial management should direct attention.[2]

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[1]This difference is institutional and political rather than economic. In the economic sense, capital has a cost to the firm whether it is supplied by owners or lenders. In the long run, suppliers of equity capital must be paid the "normal" rate of return either in the form of dividends or capital gains.

[2]Ezra Solomon, The Theory of Financial Management (New York & London: Columbia University Press, 1963) p. 8.

1. What specific assets should an enterprise acquire?

2. What total volume of funds should an enterprise commit to the acquisition of such assets?

3. How should the necessary funds be acquired?

The central problem of concern in this thesis is the latter. What insight can be gained regarding the "ideal" debt-equity ratio? Two main arguments regarding these questions have been advanced:

1. One of the traditional arguments is that the use of more debt accrues earnings to equity capital, and that the increased risk caused by using debt may not be reflected in stock prices. In this situation, the market is willing to buy more of the corporation's common stock at a higher level of risk.[3]

2. A recent argument advanced by Modigliani and Miller is that the use of more debt causes investors to require compensation for the additional risk. In this case, an increase in the debt-equity ratio results in an increase in the cost of equity capital. The decreased cost of capital by using debt is presumed to be offset by the increased cost of equity capital.[4]

3. A still more recent and generally accepted argument is that as the percentage of the debt in total financing exceeds a certain level, the financial risk also increases. As the lenders' risk increases

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[3]Arthur Stone Dewing, <u>The Financial Policy of Corporations</u> (5th ed; New York: The Ronal Press Co., 1953) pp. 836-843.

[4]Franco Modigliani and M. H. Miller, <u>The Cost of Capital, Corporation Finance, and the Theory of Investment</u> (AER, XLVII, June 1958) pp. 261-297.

and lenders impose higher rates of interest, the total cost of capital increases. Under this thesis there is some optimum point of debt equity ratio. Beyond that point the total cost of capital increases rapidly.[5]

This thesis deals with the relationship between debt financing and risk and expected rate of return under the diversification situation. This writer does not use the cost of capital approach in which the risk factor and the earning power factor are combined to determine the capitalization rate. Rather, risk and earning power are separated, evaluated and compared under alternative financing situations--equity financing and debt financing. This approach attempts to locate maximum investment utility, assuming the indifference curve represents various combinations of risk and expected rate of return which are equally satisfactory to the firm.

Definitions

Risk and Uncertainty. It is necessary to distinguish clearly between risk and uncertainty. When a set of alternative future outcomes can be assigned a definite probability distribution with confidence, the outcome situation is called "risk." When no specific probability distribution can be assigned with confidence, the outcome situation is called "uncertainty."[6] This distinction came originally from Professor Frank H. Knight. Professor Knight says:

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[5]Ezra Solomon, op. cit., pp. 91-106.

[6]Alexander A. Robichek and Steward C. Mayers, Optimal Financing Decisions (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1965) pp. 16-17.

> It will appear that a measurable uncertainty, or risk proper, as we shall use the term, is so far different from an unmeasurable one that it is not in effect an unmeasurable one, that it is not in effect an uncertainty at all. We shall accordingly restrict the term "uncertainty" to cases of the non-quantitative type.[7]

Leverage and Diversification. It is assumed in the following discussion that each unit of investment is of equal size and that the outcome of each investment is independent of the other outcomes. Therefore, when the total amount of investments is increased by using debt, two different types of effects are automatically in the total effect; that is, the "leverage effect" and the "diversification effect." The "leverage effect" is that part of the total effect which results from the change in the debt-equity ratio, and the "diversification effect" is that part of the total effect which results from the change in the investment level *per se*. When debt and equity financing are compared under the same investment situation, the *similarity* should be attributed to the diversification effect and the *difference* between them should be attributed to the leverage effect. Therefore, the definition of the leverage effect (rather than leverage) is the eventual difference in risk and expected rate of return resulting from equity financing and

[7]Frank H. Knight, *Risk, Uncertainty and Profit* (Houghton Mifflin Co., 1921) p. 20.

Assumptions

The Tax Rate. The tax factor is an institutional and a political phenomenon. It varies from country to country and from time to time. For purposes of this thesis a tax system which approximates the current U.S. system is assumed. Throughout the analysis a net income tax rate of 50 percent is used; and the interest on debt is considered deductible whereas the dividends are not.

The Interest Rate. In the first part of the argument it is assumed that the interest rate is constant at five percent, regardless of the so-called "lender's risk." Solomon comments on this point as follows:

> The trouble with this approach is that it ignores a second form of cost associated with increasing the ratio of debt to equity. This is the deterioration which increased borrowing brings about in the quality of residual net earnings, i.e., the increase in financial uncertainty. This cost is much harder to compute, but it can not be ignored.[9]

Computation Procedure

There is no mechanical problem in measuring expected rate of return. This measurement is expressed mathematically by $\frac{\Sigma Xifi}{n} / E$ where Xi is net profit after tax, fi is the probability frequency of outcome, $n = \Sigma fi$ (total frequency) and E is the amount of equity capital. More simply, the expected rate of return is the weighted average rate of return of all possible outcomes. It is also necessary to specify the measurement of risk. According to Archer and D'Ambriosio:

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[9]Ezra Solomon, op. cit., p. 80.

> The expected outcome of an investment's performance is a measure which, in general terms, indicates the center of the range of possibilities. It does not, however, tell us anything about the dispersion of the outcome from that which can be expected on the average ---. This (divergence) is what we wish to measure as risk, the extent to which an investment may turn out better or worse than expected.[10]

There are two possible measurements of dispersion. One is variance and the other is the standard deviation. The disadvantage of using the variance as a measure of dispersion is that it is not in the same units of measurements as the original data. But this disadvantage disappears when the square root of the variance is calculated, thereby expressing dispersion in terms of the standard deviation.

$$6^2 = \frac{\Sigma(x-x)^2}{N} \quad \ldots\ldots\ldots\ldots \text{variance}$$

$$6 = \sqrt{\frac{\Sigma(x-x)^2}{N}} \quad \ldots\ldots\ldots\ldots \text{standard deviation}$$

$$\frac{6}{\bar{x}} = \frac{\sqrt{\frac{\Sigma(x-x)^2}{N}}}{\bar{x}} \quad \ldots\ldots\ldots\ldots \text{coefficient of variation}$$

To maintain comparability between different sized investments, and to measure risk the coefficient of variation is used throughout this study.

A usual argument is that an increase in debt ratio in total financing leads to greater risk. The hypothesis of this thesis is that when equity financing is compared with debt financing, increased debt will lead to greater risk than equity financing. However, this study

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[10] Stephen H. Archer and Charles A. D'Ambriosio, Business Finance: Theory and Management (New York: The Macmillan Company, 1966) pp. 68-69.

maintains that not only is this tendency toward greater risk nullified under the diversification situation but an increase in expected rate of return also occurs. In other words, the increase in risk is in a sense a cost associated with the gain in the expected rate of return. The evaluation of debt financing should be determined by the comparison of these costs and gains.

Chapter II

THE VARIANCE RISK AND LEVERAGE

This section will discuss the relationship between the risk and leverage. As mentioned before, it is assumed that the probability distribution of the outcomes is given, and that risk rather than uncertainty is considered.

One Investment

This section will begin with the comparison of the following two simple cases.[1]

A. Equity financing. In the first example, suppose an investment of $1,000 has a fifty-fifty chance of making a 20 or 0 percent profit. The expected outcome or return (ER) would be (20% x 1/2 + 0% x 1/2) = 10%, and by following calculation its standard deviation is 10 percent.

Outcome %	Divergence from ER	$(D)^2$	Probability	$P \times (D)^2$
0.00	0.1	0.01	0.5	0.005
0.20	0.1	0.01	0.5	0.005
				0.010

Standard Deviation of Outcome (SR) = $\sqrt{0.01} = 0.1$

The coefficient of variation in terms of the rate of return is

(SR/ER) $\frac{0.10}{0.10} = 1$

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[1]Stephen H. Archer and Charles A. D'Ambriosio, Business Finance: Theory and Management (New York: The Macmillan Company, 1966) pp. 77-79.

B. Equity and debt financing. In the second example, suppose a similar investment exists, except that an additional $1,000 is borrowed at a rate of 5 percent. In this case, when the investment is successful, a profit of 35 percent ($2,000 x 20% - $1,000 x 5% = $350) is earned, and when the investment fails, the loss is 5 percent ($2,000 x 0% - $1,000 x 5% = -$50). The expected rate of return is (35% x 1/2 - 5% x 1/2) = 15%, and by the following calculation its standard deviation is 20 percent.

Outcome %	Divergence from ER	$(D)^2$	Probability	$P \times (D)^2$
-0.05	0.20	0.04	0.5	0.02
0.35	0.20	0.04	0.5	0.02
				0.04

The Standard Deviation of the Outcome is $\sqrt{0.04} = 0.2$

The Coefficient of Variation is $\frac{0.2}{0.15} = 1.33$

Two Investments

In the above example, the use of debt capital brings a higher expected rate of return and a higher risk than the use of equity capital, assuming the additional $1,000 borrowed capital is added to the same investment. In other words, it is invested together with the equity capital as a unit in the same indivisible investment. This is probably unrealistic.

With large investments, the borrowed capital can be used to purchase a single investment. In this case the increased safety through diversification by borrowed capital is precluded, but profitability can be increased.

When the indivisible investment is preferred to the divisible investment, there should be a better combination of risk and profitability. In the case of the above debt financing (case B), divisible investment is presumed to be available. Therefore, there are two investments, A and B, each costing \$1,000, and the probability of success of A and B is one-half, respectively. The outcome would become:

A Probability		B Probability		Total Probability		
success	1/2	success	1/2	complete success	1/4	
success	1/2	failure	1/2	half success	1/4	1/2
failure	1/2	success	1/2	half success	1/4	
failure	1/2	failure	1/2	complete failure	1/4	

Success in both investments results in a 35 percent return (\$2,000 x 20% - \$1,000 x 5% = \$350). One success and one failure results in a 15 percent return (\$1,000 x 20% - \$1,000 x 5% = \$150). When both investments fail, 5 percent is lost (\$2,000 x 0% - \$1,000 x 5% = -\$50). Then the expected outcome is \$350 (1/4) + \$150 (1/2) - \$50 (1/4) = \$150 and the rate of return to equity capital is \$150/\$1,000 = 15%. The standard deviation is \$141.42/\$1,000 = 14.142% and the risk is 0.9428.

Number of success	Earning before interest	Interest	Net profit after interest	Probability	Divergence from ER	D^2	D^2 x p
2	\$400	50	+350	1	+200	40,000	40,000
1	200	50	+150	2	0	0	0
0	0	50	-50	1	-200	40,000	40,000

Standard Deviation of the Outcome is $\sqrt{\frac{80,000}{4}} = \sqrt{20,000} = 141.421$ (14.1421%)

Expected outcome = 15%

Risk (SR/ER) = 14.1421%/15% = 0.9428.

Comparing this result with cases A and B respectively, it can be noted that this result is not only much better than case B, but also better than case A in terms of the investment indifference curve attained. That is, by borrowing an additional \$1,000 the expected outcome was raised from 10 to 15 percent, and the standard deviation changed from 0.1 (10% of equity capital) to 0.14142 (14.142% of equity capital = \$1,000). As a result, risk decreased from 1 to 0.9428 by borrowing debt capital under the divisible situation. This situation is shown in Figure 1.

As illustrated in Figure 1, when the indivisible investment is financed by debt, both the expected rate of return and the risk increase (from A to B), but when the investment is divisible, the debt financing increases the expected rate of return and at the same time decreases risk (from A to C). Point C is absolutely superior to point A.

As a next step it is assumed that there are two investments of \$1,000 each and \$1,000 equity capital is available: the decision is whether equity or debt should be used to finance the other investment. In this case the tax effect and the depreciation effect must be considered. The income tax rate is assumed to be 50 percent and the annual depreciation cost is 5 percent of the original asset cost.

A. <u>The short run effects</u>. As illustrated in Table 1, the use of equity capital results in a one-fourth probability of getting \$200 profit (after tax) and one-half probability of getting \$100 profit (after tax) and one-fourth probability of getting \$0 profit (after tax). Then the expected rate of return is five percent (ER = \$100/\$2,000 = 5%) and

A Comparison of Divisible Investments and Indivisible Investments.

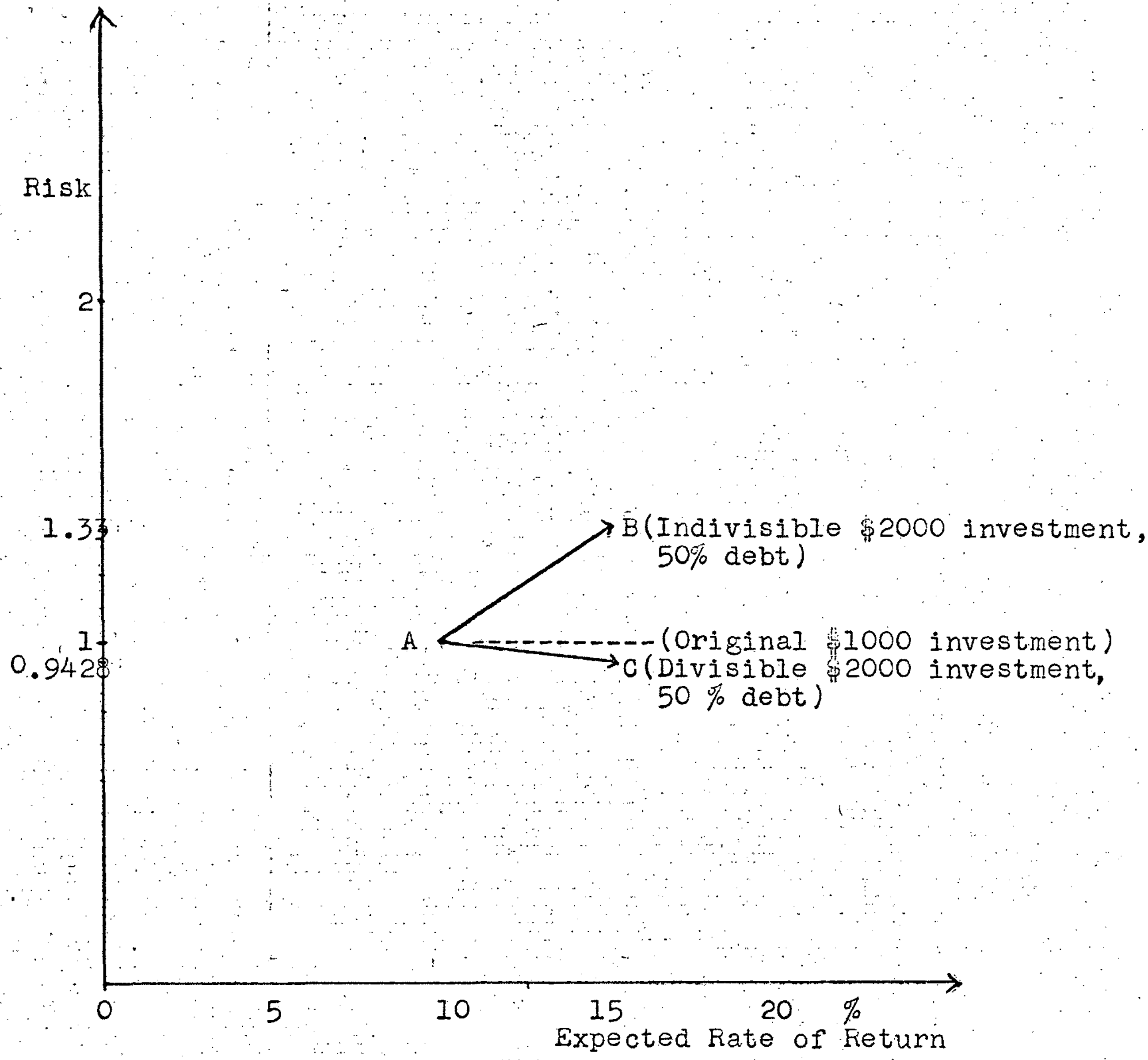

Figure 1

the coefficient of variation is 3.535 percent (SR = \$70.71/\$2,000 = 0.03535 = 3.535%) with the risk equal to $\frac{3.535}{5}$ = 0.707.

Dependence on debt capital (Table 2) necessitates paying interest whether or not a profit is made. Then the net profit after interest is, assuming successes in both investments, \$400 - \$50 = \$350; success in only one investment, \$200 - \$50 = \$150; and failure in both investments, \$0 - \$50 = -\$50.

When the profit before tax is -\$50, there is no tax, but any profit is taxed at 50 percent. The net profit after tax is respectively \$175, \$75, -\$50 (column 5 of Table 2). Using the same procedure as for equity capital, the expected return is x = 75 - $\frac{25}{4}$ = 68.75 (column 8) and the standard deviation is σ = 80.0. From the viewpoint of equity capital, the expected rate of return ER = \$68.57 = 6.875%. As a result the risk is 8.00%/6.875% = 1.163.[2]

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[2]The expected return X is obtained from Table 2 in the following way:
$\bar{X}$ = 75 (temporary mean, column 5) - $\frac{25}{4}$ (adjustment, from column 8) = 68.75.
This is eventually the same thing as following calculation (columns 5 & 7)

$$\bar{X} = \frac{\Sigma X_i P_i}{n} = \frac{175 \times 1 + 75 \times 2 - (-50) \times 1}{4}$$

$$= \frac{175 + 150 - 50}{4} = \frac{275}{4} = 68.75$$

Standard Deviation σ is obtained from column 9, Table 2.

$$\sigma = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n}} = \sqrt{\frac{25.625}{4}} = 80$$

This is expressed in terms of percentage of the investment size.

$$\sigma = \frac{\$80}{\$1,000} = 8.00\%$$

TABLE 1

MEASUREMENT OF EXPECTED RATE OF RETURN AND RISK FOR TWO INVESTMENTS (100% EQUITY)

In the short run

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax	Diver-gence $u=x-\bar{x}$	Proba-bility p	U x P	U^2 x P
2	400	0	400	200	+100	1	+100	10,000
1	200	0	200	100	0	2	0	--
0	0	0	0	0	-100	1	-100	10,000
Total						4	0	20,000

TABLE 2

DO. (50% EQUITY AND 50% DEBT)

In the short run

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax	Diver-gence $u=x-\bar{x}$	Proba-bility p	U x P	U^2 x P
2	400	50	350	175	100	1	100	10,000
1	200	50	150	75	0	2	0	0
0	0	50	-50	-50	-125	1	-125	+15,625
Total						4	-25	25,625

Table I

$\bar{x} = \$100/\$2{,}000 = 5\%$

$\delta = \sqrt{\frac{20{,}000}{4}}/\$2{,}000 = \$70.71/\$2{,}000 = 3.535\%$

Risk $= \frac{\delta}{\bar{x}} = 3.535/5 = 0.707$

Table 2

$\bar{x} = \$68.75/\$1{,}000 = 6.875\%$

$\delta = \sqrt{\frac{25{,}625}{4}}/\$1{,}000 = \$80.0/\$1{,}000 = 8.00\%$

Risk $= \frac{\delta}{\bar{x}} = 8.00/6.875 = 1.163$

TABLE 3

MEASUREMENT OF EXPECTED RATE OF RETURN AND RISK
FOR ONE INVESTMENT (100% EQUITY)

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax	Divergence $u=x-\bar{x}$	Probability p	U x P	U^2 x P
1	200	0	200	100	+50	1	+50	2,500
0	0	0	0	0	-50	1	-50	2,500
Total						2	0	5,000

$\bar{x} = 50 \qquad ER = 50/1000 = 5\%$

$\sigma = \sqrt{\frac{5000}{2}} = 50.00 \qquad \sigma = 50.00/1000 = 5.000\%$

$\text{Risk} = \frac{\sigma}{ER} = \frac{5.000}{5} = 1.000$

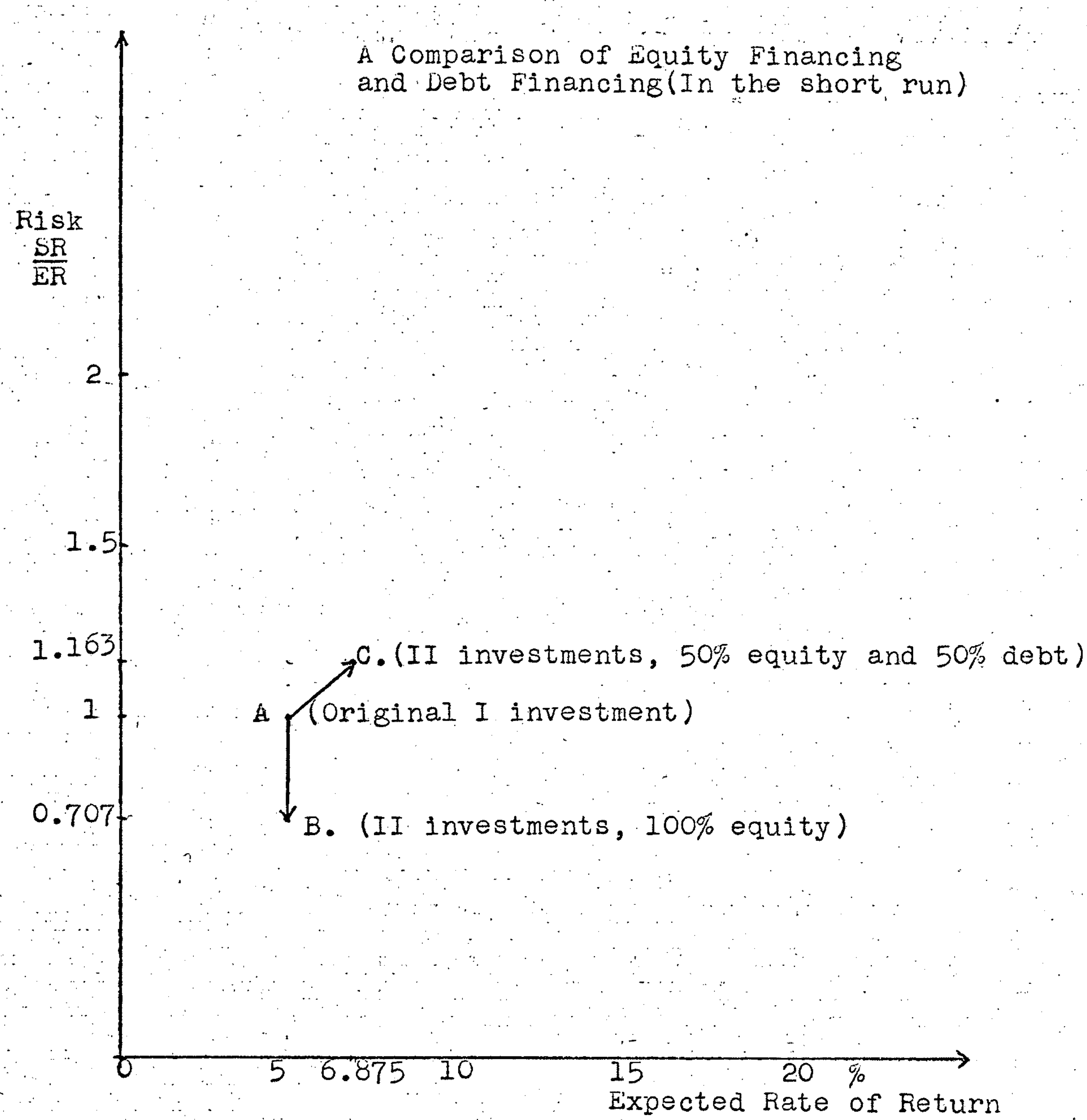
A Comparison of Equity Financing
and Debt Financing(In the short run)
Risk
SR/ER
2
1.5
1.163
1
0.707
C.(II investments, 50% equity and 50% debt)
A (Original I investment)
B. (II investments, 100% equity)
0
5
6.875
10
15
20
%
Expected Rate of Return

Figure 2

In connection with the short run risk of debt capital, it is important to consider the effect of depreciation allowances. Generally, as the number of investments increases, the total depreciation allowance increases, but the risk of running out of cash in the short run decreases conspicuously even if debt capital is used.

A hypothetical case is assumed where equity capital is not used at all. In this example annual depreciation allowances are assumed to be 5 percent of the original cost of the purchased assets and the greatest loss occurs when every investment results in complete failure; that is, earnings are zero. The amount of interest depends on the total amount of debt which is, in this case, equal to the amount of total investment.

As illustrated in Table 4, as the number of investments increase, both the amount of the maximum loss and the depreciation allowance increase at the same rate, but the probability of the greatest loss decreases conspicuously as the number of investments increase. This can be explained as follows: when there are three investments, the probability of greatest loss is $1/2^3 = 1/8$ because each investment has a 50 percent probability of failure. In the same way the probability of the maximum loss is $1/2^{10} = 1/1024$ for ten investments.

When equity capital is used, at least one unit, then depreciation less the loss is greater than zero $(B - A > 0)$, because when the equity investment fails, the loss is \$0, but the depreciation allowance is \$50.

From this discussion it can be said that even if each investment has a .5 probability of failure, the short-run risk is decreased conspicuously by borrowing more, assuming diversification. Of course,

TABLE 4

THE EFFECT OF DEPRECIATION ALLOWANCES ON SHORT-RUN RISK

Number of investment	1	2	3	4	5	6	7	8	9	10
Probability of greatest loss	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{256}$	$\frac{1}{512}$	$\frac{1}{1024}$
A Amount of greatest loss	50	100	150	200	250	300	350	400	450	500
B Depreciation	50	100	150	200	250	300	350	400	450	500
B - A	0	0	0	0	0	0	0	0	0	0

this situation is applicable to equity capital too.

B. <u>The long run effects</u>. So far the short-run (one year) situations have been discussed. And it was found that there is no need to worry about the short-run risk. But in the long run, even if liquidity is maintained by depleting depreciation allowances, there are serious consequences.

The major difference between the long-run and the short-run situation is that in the long run the loss in one period is not only exempted from taxes, but if the loss exceeds current profits, it can be deducted from the profit of following years. Thus, in the long run the tax is levied on the income remaining after the deduction of all previous losses: the loss strengthens the tax position.

These facts are reflected in Table 5. The expected rate of return is 7.5% ($75/$1,000), the standard deviation is 7.071% ($70.71/$1,000) and the risk is 0.9428 (7.071%/7.5%). This situation can best be explained by Figure 4. The risk and the expected outcome with 100 percent equity financing do not change between the long and short run because the use of equity never shows a loss. The same is true even if the tax effect is considered.

When both investments are compared in the long run, the situation can be improved by using debt capital with divisible investments. The point changes from C to D; that is, risk decreases from 1.163 to 0.9428 at the same time the expected outcome increases from 6.875% to 7.5%. Then the decision is between points B and D. The choice between points B and D depends chiefly on the attitude of the investor. Generally an

TABLE 5

MEASUREMENT OF EXPECTED RATE OF RETURN AND RISK FOR TWO INVESTMENTS (50% EQUITY AND 50% DEBT)

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax	Divergence $u=\frac{x-\bar{x}}{100}$	Probability p	U x P	U^2 x P
2	400	50	350	175	+1	1	1	1
1	200	50	150	75	0	2	0	0
0	0	50	-50	-25*	-1	1	-1	1
Total						4	0	2

*The difference with Table 2 is only this part.

$\bar{x} = 75$

$\delta = 100\sqrt{\frac{2}{4}} = 100\sqrt{\frac{1}{2}} = 100\sqrt{0.5} = 100 \times 0.7071 = 70.71$

ER = \$75/\$1000 = 7.5%

SR = \$70.71/\$1000 = 7.071%

Risk $= \frac{SR}{ER} = \frac{7.071}{7.5} = 0.9428$

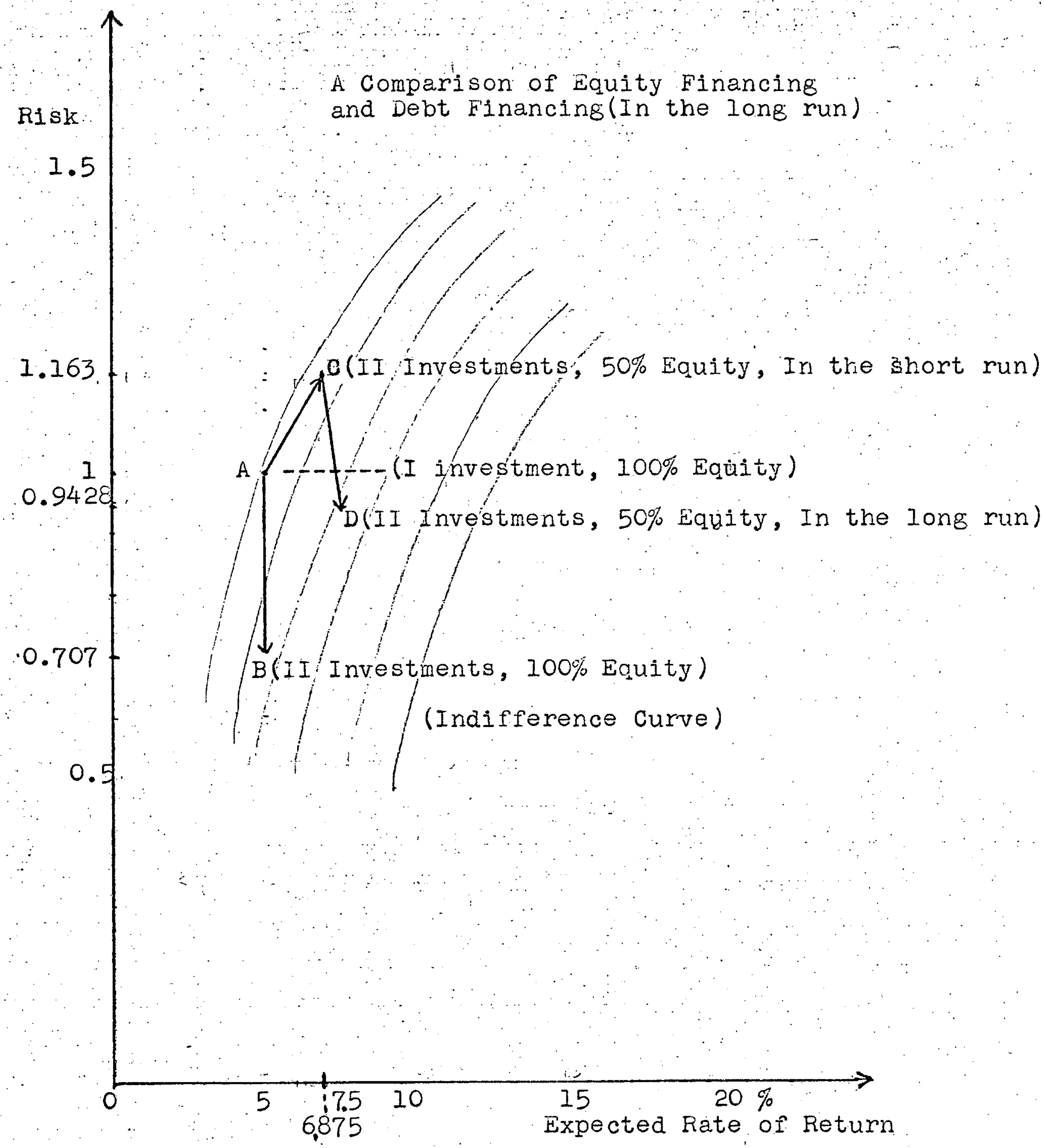
A Comparison of Equity Financing
and Debt Financing(In the long run)
Risk
1.5
1.163
1
0.9428
0.707
0.5
C(II Investments, 50% Equity, In the short run)
A
(I investment, 100% Equity)
D(II Investments, 50% Equity, In the long run)
B(II Investments, 100% Equity)
(Indifference Curve)
0
5
7.5
6.875
10
15
20 %
Expected Rate of Return

Figure 4

aggressive investor would prefer the point D to the point B. This situation can be more clearly explained by considering the following extreme case.

Graphically, the effect of diversification is vertically downward, and the effect of leverage is horizontal. Therefore, the use of debt capital in diversified situations produces both a leverage and a diversification effect (risk decreasing effect), whereas the use of equity financing in diversified situations does not improve earnings, but only decreases risk.[4]

The comparison between debt and equity financing in terms of investment indifference curves reveals that the use of more debt changes the point from northwest to southeast; that is, it moves along the shortest distance to the higher indifference curve, whereas the use of debt financing decreases risk but does not increase profitability; therefore, the direction of the move is not the shortest cut to a higher indifference curve.

Extreme Cases:

A. Equity financing. Assume an investment in ten different projects, each of which costs $1,000 and has a fifty-fifty probability

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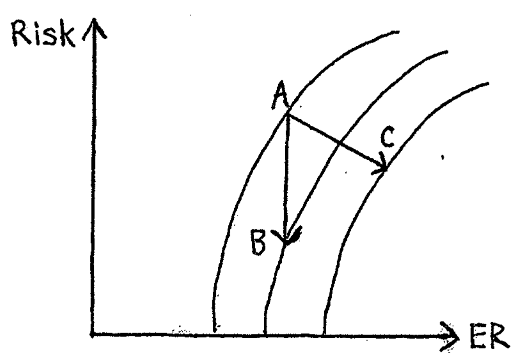
4



Diagramatically, when the distance AB equal to AC, the higher investment indifference curve can be reached by AC rather than AB. This direction A - C is the shortest cut to higher investment indifference curve.

of returning 20 percent or 0 percent. Each investment is statistically independent. The probability of ten successes out of ten projects will be $_{10}C_{10}(1/2)^{10}\ (1/2)^{0} = 1/1024$, the probability of nine successes out of ten will be $_{10}C_{9}(1/2)^{9}\ (1/2)^{1} = 10/1024$ and so on, as shown in Table 6.

This table indicates that average expected rate of return after taxes is 5 percent ($500/$10,000), the standard deviation is 1.58 percent ($1.58/$10,000) and the risk is 0.32 (1.58/5).

B. Debt financing. In this case, assume that the first unit of investment requires equity capital and that all additional sources of money are obtained only through debt financing. Furthermore, assume that the rate of interest is 5 percent. It can be seen from Table 7 that the expected rate of return is 27.5 percent ($275/$1,000), the standard deviation is 15.8 percent ($158/$1,000), and the risk is 0.57 percent (15.8/27.5). The probability that the profit is less than zero (a loss) is 1.74 (Z = $27.5 - $0/15.8 = 1.74 standard deviation) and from the table of a normal curve area, it is 0.4591. The probability of incurring a loss is 4.09 percent (0.5 - 0.4591).

A comparison of the two examples is illustrated in Figure 5. Point M on the higher investment indifference curve is preferable to point K, showing the investors' preference to debt financing. The use of equity capital does not improve the expected rate of return; it only decreases risk. Debt capital, however, increases the expected rate of return, also decreasing the risk.

TABLE 6

MEASUREMENT OF EXPECTED RATE OF RETURN AND RISK FOR TEN INVESTMENTS (100% EQUITY)

In the long run
In the short run

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax(x)	Probability p	Divergence $U=\frac{x-\bar{x}}{100}$	P x U	P x U^2
10	2,000	0	2,000	1,000	1	5	5	25
9	1,800	0	1,800	900	10	4	40	160
8	1,600	0	1,600	800	45	3	135	405
7	1,400	0	1,400	700	120	2	240	480
6	1,200	0	1,200	600	210	1	210	210
5	1,000	0	1,000	500	252	0	0	0
4	800	0	800	400	210	-1	-210	210
3	600	0	600	300	120	-2	-240	480
2	400	0	400	200	45	-3	-135	405
1	200	0	200	100	10	-4	-40	160
0	0	0	0	0	1	-5	-5	25
Total					1,024	0	0	2,560

$\bar{x} = 500$

$\delta = 100\sqrt{\frac{2560}{1024}} = 100 \times 1.58 = 158$

The Expected Rate of Return = \$500/\$10,000 = 5%

Standard Deviation = \$158/\$10,000 = 1.58%

Risk = 1.58/5 = 0.32

TABLE 7

MEASUREMENT OF EXPECTED RATE OF RETURN AND RISK FOR TEN INVESTMENTS (10% EQUITY, 90% DEBT)

In the long run

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax (x)	Divergence $U = \frac{x - \bar{x}}{100}$	Probability p	U x P	U^2 x P
10	2,000	450	1,550	775	5	1	5	25
9	1,800	450	1,350	675	4	10	40	100
8	1,600	450	1,150	575	3	45	135	405
7	1,400	450	950	475	2	120	240	480
6	1,200	450	750	375	1	210	210	210
5	1,000	450	550	275	0	252	0	0
4	800	450	350	175	-1	210	-210	210
3	600	450	150	75	-2	120	-240	480
2	400	450	- 50	- 25	-3	45	-135	405
1	200	450	- 250	-125	-4	10	- 40	160
0	0	450	- 450	-225	-5	1	- 5	25
Total					0	1,024	0	2,560

$\bar{x} = 275$

$$\sigma = 100\sqrt{\frac{2560}{1024}} = 100 \times \sqrt{2.5} = 100 \times 1.58 = 158$$

The Expected Rate of Return = \$275/\$1000 = 27.5%

Standard Deviation of Outcome = \$158/\$1000 = 15.8%

Risk = 15.8%/27.5% = 0.57

A Comparison of Equity Financing
and Debt Financing(Ten Investments)

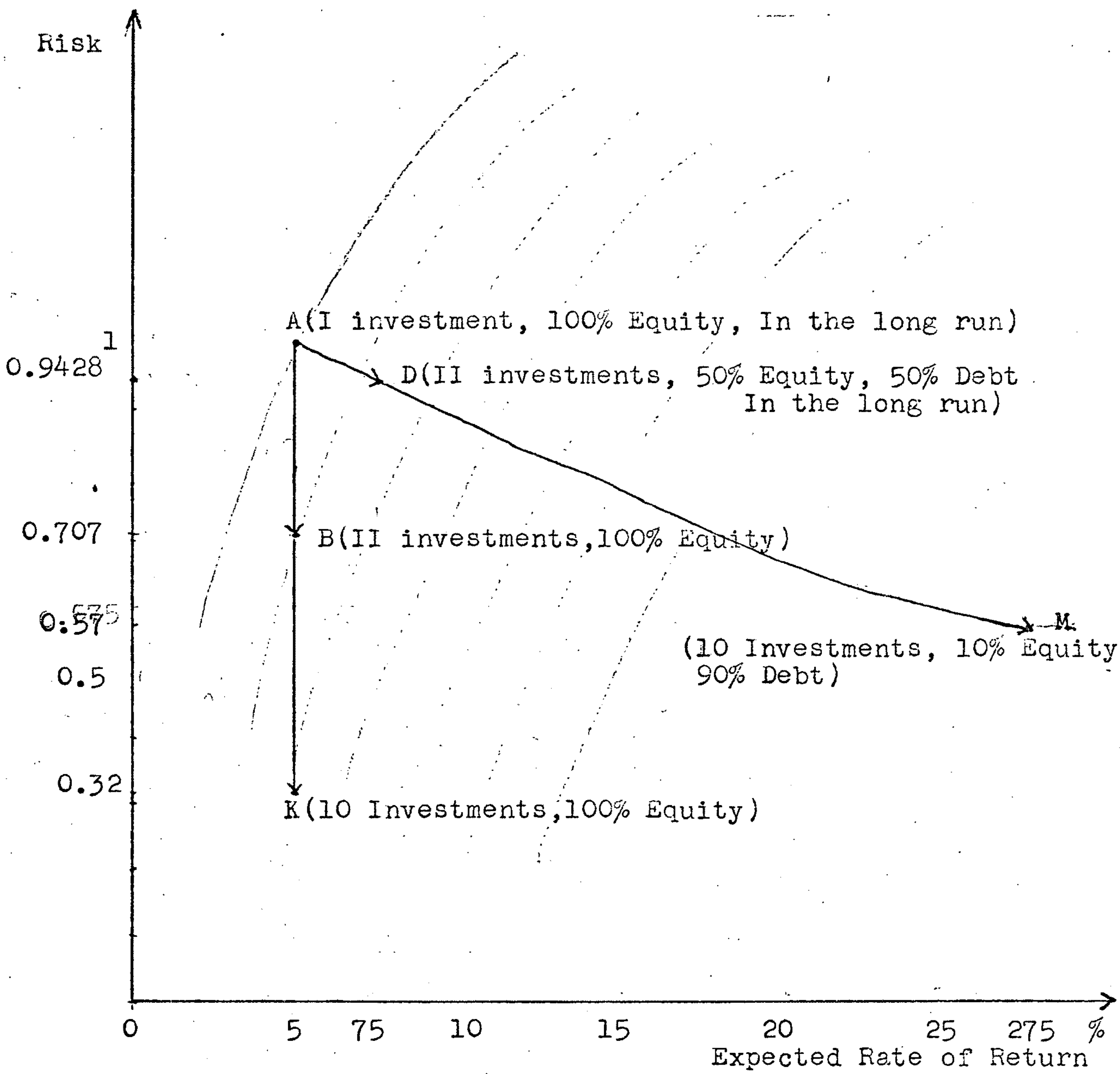


Figure 5

Lender's Risk

A generally accepted principle in finance is that as the percentage of the debt in total financing becomes larger the lender's risk also increases. As a result, lenders request higher rates of interest to compensate for the increased risk and accordingly, the cost of capital increases.[5] But insofar as the above examples are concerned, the lender's risk decreases as the percentage of the debt in total financing increases.

By definition, lender's risk is the risk that the lenders will not receive either principal or interest or both. There is no lender's risk if no debt is assumed. Lender's risk appears only when at least one unit of debt capital is used. Table 5 reveals that the probability that the lenders will not be repaid their principal is one-fourth, and the total amount of their loss is $500 ($25 x 20 years). The expected loss is calculated in the same way as the expected rate of return on page 11; that is $125 (1/4 x $500).

The following expected loss can be computed from Table 7 when one unit of equity and nine units of debt are used:

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[5]Ezra Solomon, "Leverage on the Cost of Capital." Foundations for Financial Management. A Book of Readings. by James Van Horne. The quotation is from p. 409.

> "But in practice, ki (cost of debt capital), the average rate of interest paid on debt, must rise as leverage is increased. For extreme leverage positions, i.e., as the company approaches an all-debt situation, it is clear that ki will be at least equal to ko (cost of overall capital). Given the general attitude of bondholders and bond rating agencies, it is highly likely that ki will be above ko for positions of extreme leverage."

Total loss	Probability	Expected loss
\$25 x 20 years = \$500	45/1,024	22,500/1,024
\$125 x 20 years = \$2,500	10/1,024	25,000/1,024
\$225 x 20 years = \$4,500	1/1,024	4,500/1,024
		52,000/1,024

\$52,000/1,024 = \$50.

The comparison of these two examples indicates that the expected loss decreases conspicuously from \$125 to \$50 with an increase in debt from 1 unit to 9 units. But both cases do not cause any loss as far as the \$1,000 equity capital is concerned. This is the entrepreneur's and not the lender's risk. The lender's risk is only the deficit less the equity capital; in both cases the interest is already paid. It can therefore be concluded from the above discussion that lender's risk decreases with more debt financing. Theoretically, the corporation can require lenders to cut down the rate of interest because of the lender's increased safety.

The Effect of a Change in Variance

In the above example it was assumed that success resulted in a 20 percent return and failure resulted in a 0 percent return. Although the average of the rate of return is 10 percent, the total standard deviation doubles when the standard deviation of each investment is doubled. Therefore, the risk is doubled. For example, suppose an investment has a fifty-fifty chance of yielding a 40 percent profit or a 20 percent loss. The expected rate of return is 10 percent, but the standard deviation and the risk become three times what they were in the original example.

Depending on the nature of the industry, different standard deviations can be applied; but the validity of this approach in comparing a company using higher proportions of debt and a company using higher proportions of equity does not change, because both companies are assumed to be identical except in their capital structures. There is no change in the standard deviation because of a difference in the capital structures. This situation is illustrated in Tables 8 and 9. As the proportion of equity capital increases, the relative standard deviation applicable to the equity capital decreases.

The Effect of a Change in the Rate of Expected Return

As Figure 6 and Table 10 illustrate, the risk situation changes as the expected rate of return (before taxes and interest) goes down. With a 10 percent expected rate of return, the effect of diversification is much larger than the leverage effect. In the case of one investment, the risk is 1.000; but by borrowing one unit more under diversification the risk decreases to 0.707. If an additional nine units of debt capital are added, the risk decreases to 0.575.

When the expected rate of return goes below 8.5 percent, the entrepreneur's risk of using debt capital exceeds the risk of the original one unit equity investment. With 90 percent debt financing and 10 percent equity financing, the entrepreneur's risk reaches infinity as the expected rate of return reaches 4.5 percent (5% x 90%); any rate below 4.5 percent makes the lender's risk a reality. Similarly, one unit of equity capital and one unit of debt capital increase the entrepreneur's risk infinitely as the expected rate of return

TABLE 8

MEASUREMENT OF EXPECTED RATE OF RETURN AND RISK FOR ONE INVESTMENT (100% EQUITY)

Number of success	Earning	Interest	Net profit after tax	Divergence $U=x-\bar{x}$	Probability p	U x P	U^2 x P
1	400	0	200	+150	1	+150	22,500
0	-200	0	-100	-150	1	-150	22,500
Total				0	2		45,000

The Expected Rate of Return ER = \$50/\$1000 = 5%

The Standard Deviation of the Outcome $\delta = \sqrt{\frac{45,000}{2}}$ /\$1000 = \$150.0/\$1000 = 15.00%

Risk = δ/ER = 15.00/5 = 3.000

This risk is three times that of Table 3.

TABLE 9

MEASUREMENT OF THE EXPECTED RATE OF RETURN AND RISK
FOR ONE INVESTMENT (100% DEBT)

Number of success	Earning	Interest	Net profit after tax	Divergence $U=x-\bar{x}$	Probability p	U x P	U^2 x P
1	400	50	175	150	1	+150	22,500
0	-200	50	-125	-150	1	-150	22,500
Total				0	2		45,000

The Expected Rate of Return ER = \$25/\$1000 = 25%

The Standard Deviation of the Outcome $\sigma = \sqrt{\frac{45,000}{2}} / 0 = +\infty$

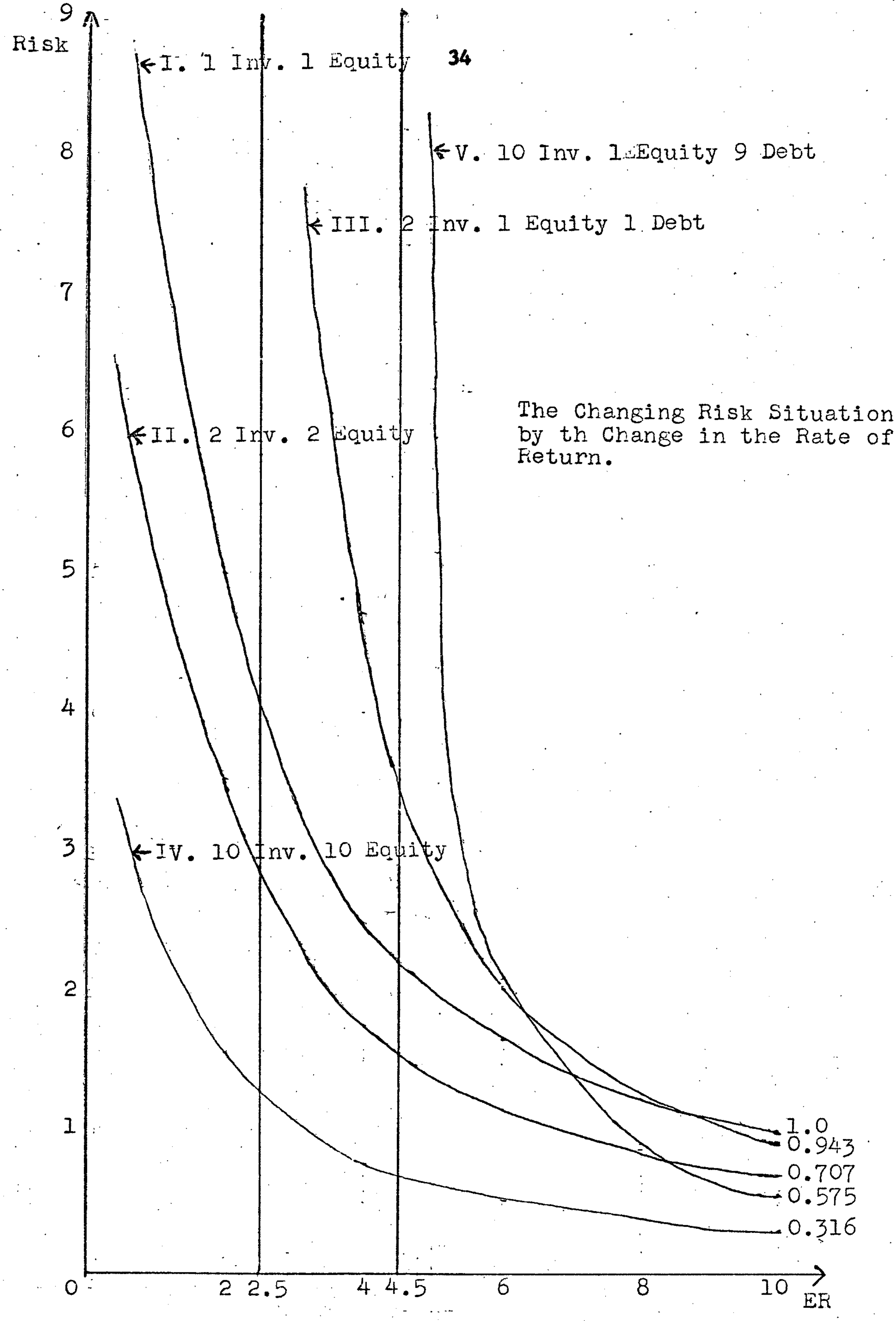
Risk
I. 1 Inv. 1 Equity
V. 10 Inv. 1 Equity 9 Debt
III. 2 Inv. 1 Equity 1 Debt
II. 2 Inv. 2 Equity
IV. 10 Inv. 10 Equity
The Changing Risk Situation by th Change in the Rate of Return.
1.0
0.943
0.707
0.575
0.316
0
1
2
3
4
5
6
7
8
9
2
2.5
4
4.5
6
8
10
ER

Figure 6

TABLE 10

THE CHANGING RISK SITUATION BY THE CHANGE IN THE RATE OF RETURN

Complete Success	20	18	16	14	12	10
Average Earning Rate	10	8	6	4	2	0
Complete Failure	0	-2	-4	-6	-8	-10
I. One Investment - 100% Equity						
After Tax Profit (P)	5	4	3	2	1	0
Standard Deviation(δ)	5.0	5.0	5.0	5.0	5.0	5.0
Risk = δ/P	1.0	1.25	1.666	2.5	5.0	0.0
II. Two Investments - 100% Equity						
After Tax Profit (P)	5	4	3	2	1	0
Standard Deviation(δ)	3.535	3.535	3.535	3.535	3.535	3.535
Risk = δ/P	0.707	0.884	1.178	1.767	3.535	0.0
III. Two Investments - 50% Equity, 50% Debt						
After Tax Profit (P)	7.5	5.5	3.5	1.5	-0.5	-2.5
Standard Deviation(δ)	7.071	7.071	7.071	7.071	7.071	7.071
Risk = δ/P	0.943	1.285	2.020	4.714	(-14.142)	(-2.828)
IV. Ten Investments - 100% Equity						
After Tax Profit (P)	5	4	3	2	1	0
Standard Deviation(δ)	1.58	1.58	1.58	1.58	1.58	1.58
Risk = δ/P	0.316	0.395	0.527	0.79	1.58	0.0
V. Ten Investments - 10% Equity, 90% Debt						
After Tax Profit (P)	27.5	17.5	7.5	-2.5	-12.5	-22.5
Standard Deviation(δ)	15.8	15.8	15.8	15.8	15.8	15.8
Risk = δ/P	0.575	0.903	2.107	(-6.32)	(-1.274)	(-0.702)

reaches 2.5 percent; when the rate goes below 2.5 percent the lender's risk is apparent.

In the case where debt capital is not used at all, the lender's risk does not exist, but the entrepreneur's risk increases infinitely as the expected rate of return approaches 0. Therefore, the potential lender's risk is apparent when the expected rate of return drops below 0 percent. In any case, absolute security can not be obtained in the world of business where risk is always inherent.

Chapter III

CONSTANT AND CHANGING INTEREST RATES

Re-examination of the Constant Interest Rate

In the previous discussion, the interest rate was assumed to be a constant 5 percent regardless of the ratios of debt to equity, which may be unrealistic. Therefore, it would be useful to examine the consequences when the interest is assumed to vary. In this case the first lender is assumed to have prior right to a specified amount of principal in the event of liquidation. In the same way the second lender has priority over the third lender and so forth. Therefore, the last lender's risk is greater than the second-to-last lender's risk and always the nth lender's risk is greater than the n-1th lender's risk.

As a matter of fact, as the lender's risk becomes larger, the interest rate which they charge becomes higher. But the important fact is that the risk of the additional n+1th lender is greater than the risk of the current nth lender. If a constant ratio between the lender's risk and the interest rate is assumed, then the interest rate is doubled when the lender's risk is doubled.

However, the measurement of risk is one of the most difficult problems. The lender's risk was previously defined as the probability that the lender will not be repaid either principal or interest or both. For simplicity of calculation, the same probability distribution is assumed every year. Therefore, when the mean earnings during the first year are $68.75, the total earnings for 20 years are $1,375

(\$68.75 x 20 years), and when the standard deviation of the first year is \$80.00, the total standard deviation for 20 years is \$1,600 (\$80.00 x 20 years). If the earnings are retained at the end of the 20th year the total equity capital should be \$2,375 (original equity \$1,000 + retained earnings \$1,375). Then the probability that the total value of equity becomes less than zero can be obtained by the following computation:

$$Z = \frac{2375 - 0}{1600} = 1.4843 \text{ (standard deviation)}$$

(from normal curve area table) 0.4306 --- 43.06%

0.5 - 0.4306 = 0.0694 --- 6.94%

Therefore, the risk of the first lender is 6.94 percent. This situation is illustrated in Case 1.

As long as a dividend policy is maintained where dividends are paid only after the payment of interest, and only when a profit is realized, then a change in dividend policy does not effect the lender's risk. The corporation will not pay a dividend when net profits after tax are less than zero. Therefore, the absolute amount of standard deviation decreases annually as long as they pay yearly dividends.

As illustrated in Case 2, the use of one more unit of debt increases the risk of the lender from 6.94 percent to 7.21 percent, and the risk of the first lender decreases from 6.94 percent to 2.62 percent. But the important comparison is the risk of the first lender in Case 1 and the risk of the second lender in Case 2. If the risk of the latter is larger than the risk of the former, the second lender will charge a higher interest rate. In this case the risk of the second lender in

Case 2 (7.21%) is more than the risk of the first lender in Case 1 (6.94%). Therefore, the second lender will charge more than the first lender--that is, 5.2 percent. However, the risk of the last lender in Case 3 is 6.40 percent and this is smaller than both the risks of the last lender in Case 2 (7.21%) and the last lender in Case 1 (6.94%). Also, when junior debt is acquired, the risk of the senior lender decreases rapidly.

Two units of debt make the first lender's risk 2.62 percent and three units of debt decrease the first lender's risk to 0.73 percent. The corporation can demand a lower interest rate from the first lender, because his security is increased by the increase of the junior debt. But there are certain disadvantages underlying this theory.

First, economic fluctuation is assumed to be negligible; in other words business risk is disregarded and the discussion is limited to financial risk. However, viewing the absence of any serious economic depression since 1930 and the general upward trend of the economy, the above example can be interpreted more favorably for debt financing. Secondly, the average earning rate is assumed to be 10 percent, but this rate may be decreased as the number of investments increases. Lastly, the model presented is only applicable to economies where free competition exists.

Changing Interest Rate

In the previous section the assumption that interest rates are constant regardless of the ratio of debt to equity was examined. This section will analyze and build a model which would include increasing interest rates. One of the basic problems is to attach a numerical value to these changing interest rates. It is certain that when the lender's risk increases, interest rates tend to increase. But the lender's risk can not be considered independently of business risk since it is not usually forecasted with any numerical precision. For this reason interest rates of 5, 5.25, and 5.5 percent...7.00% are arbitrarily assigned to the first, second, third,...ninth lender respectively. The situation is then analyzed in Cases 6, 7 and 8. Table 11 provides a comparison of the first and second sections of this chapter.

Table 11 shows that when the lender determines the interest rate according to his risk (in this case the lenders are assumed to demand at least a 5 percent interest rate regardless of the risk), each additional lender's risk except the second lender's risk goes down. The last lender's risk (9th lender) is as low as 2.75 percent. In the same way, when increasing interest rates are assumed, the additional lender's risk goes down but the decreasing rate is not as conspicuous as it was in the former case. This indicates that the lender's risk will increase if the interest rate increases rapidly, which in turn, suggests that the increasing rate of interest is not the result, but the cause, of the increasing risk of lenders.

CASE 1

Equity:1
Debt:1

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax (x)	Divergence $U=x-\bar{x}$	Probability p	UxP	U^2xP
2	400	50	350	175	100	1	100	10.000
1	200	50	150	75	0	2	0	0
0	0	50	-50	-50	-125	1	-125	15.625
Total					-25	4	-25	25.625

$\bar{x}$ = \$68.75 $\qquad$ $\bar{x}$ = 68.75 x 20 = 137.5

σ = \$80.00 $\qquad$ σ = 80.00 x 20 = 1600

$\bar{x}$ + E = 1375 + 1,000 = 2,375

$$z = \frac{2,375 - 0}{1,000} = 1.48 \text{ - - - - - } 0.4306 \text{ - - - - - } 43.06\%$$

The risk of the first lender - - - - - 50% - 43.06% = 6.94%

CASE 2

Equity:1
Debt:2

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax (x)		Diver-gence $U = \frac{x - \bar{x}}{50}$	Proba-bility p	UxP	U^2xP
3	600	100	500	250	150	3	1	3	9
2	400	100	300	150	50	1	3	3	3
1	200	100	100	50	-50	-1	3	-3	3
0 0	0	100	-100	-700	-200	-4	1	-4	16
Total						-1	8	-1	31

$\bar{x} = 100 - 1/8 \times 50 = 100 - 6.25 = 93.75$
$\sigma = 50\sqrt{31/8} = 50\sqrt{3.875} = 50 \times 1.97 = 98.5$

$\bar{x} = 93.75 \times 20 = 1875$
$\sigma = 98.5 \times 20 = 1970$
$\bar{x} + E = 1{,}875 + 1{,}000 = 2{,}875$

$Z = \frac{2{,}875 - 0}{1{,}970} = 1.46$ - - - - - 0.4279 - - - - - 42.79%

The risk of the second lender - - - - - 50% - 42.79% = 7.21%
The risk of the first lender - - - - - $Z = \frac{2{,}875 + 1{,}000}{1{,}970} = 1.94$ - - - - - 0.4738
50% - 47.38% = 2.62%

CASE 3

Equity:1
Debt:3

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax (x)	Divergence $U=\frac{x-\overline{x}}{25}$	Probability p	UxP	U^2xP
4	800	152	648	324	8	1	8	64
3	600	152	448	224	4	4	16	64
2	400	152	248	124	0	6	0	0
1	200	152	48	24	-4	4	-16	64
0	0	152	-152	-152	-11	1	-11	121
Total					-3	16	-3	313

$\overline{x} = 124 - 3/16 \times 25 = 124 - 4.68 = 119.32$
$\sigma = 25\sqrt{313/16} = 25\sqrt{19.56} = 25 \times 4.42 = 110.50$

$\overline{x} = 119.32 \times 20 = 2{,}386$
$\sigma = 110.50 \times 20 = 2{,}210$
$\overline{x} + E = 2{,}386 + 1{,}000 = 3{,}386$

$Z = \frac{3.386}{2.210} = 1.53$ - - - - - 0.4370 - - - - - 43.7%

The risk of the last lender - - - - - 50% - 43.7% = 6.4%

The risk of the first lender - - - - - $Z = \frac{3.386 \quad 2.000}{2.210} = \frac{5.386}{2.210} = 2.44$ - - - - - 49.27%

50% - 49.27% = 0.73%

CASE 4

Equity:1
Debt:4

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax (x)	Divergence $U = \frac{x-\bar{x}}{50}$	Probability p	UxP	U^2xP
5	1000	202	798	399	5	5	5	25
4	800	202	598	299	3	5	15	45
3	600	202	398	199	1	10	10	10
2	400	202	198	99	-1	10	-10	10
1	200	202	-2	-2	-3	5	-15	45
0	0	202	-202	-202	-7	1	-7	49
Total					-2	32	-2	184

$\bar{x} = 149 - 2/32 \times 50 = 149 - 3.12 = 145.88$

$\sigma = 50\sqrt{184/32} = 50\sqrt{5.75} = 50 \times 2.40 = 120$

$\bar{x} = 145.88 \times 20 = 2917.6$

$\sigma = 120 \times 20 = 2400$

$\bar{x} + E = 2,918 + 1,000 = 3,918$

$z = \frac{3,918 - 0}{2,400} = 1.63$ - - - - - 0.4484 - - - - - 44.84%

The risk of the last lender - - - - - 50% - 44.84% = 5.16%

CASE 5

Equity:1
Debt:9

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax (x)	Divergence $U=\frac{x-\overline{x}}{100}$	Probability p	UxP	U^2xP
10	2000	450	1550	775	5	1	5	25
9	1800	450	1350	675	4	10	40	100
8	1600	450	1150	575	3	45	135	405
7	1400	450	950	475	2	120	240	480
6	1200	450	750	375	1	210	210	210
5	1000	450	500	275	0	252	0	20
4	800	450	350	175	-1	210	-210	210
3	600	450	50	75	-2	120	-240	480
2	400	450	-50	-50	-3.25	45	-146.25	475.35
1	200	450	-250	-250	-5.25	10	-52.5	275.62
0	0	450	-450	-450	-7.25	1	-7.25	52.56
Total					-3.75	1024	-26	2773.53

$\overline{x} = 275 - \frac{26}{1024} \times 100 = 275 - 2.54 = 272.46$

$\overline{x} = 272.46 \times 20 = 5,449.2$

$\sigma = 100\sqrt{\frac{2774}{1024}} = 100\sqrt{2.709} = 100 \times 1.646 = 164.6$

$\sigma = 164.6 \times 20 = 3292.0$

$\overline{x} + E = 1,000 + 5,449.2 = 6,449.2$

$Z = \frac{6,449 - 0}{3,292} = 1.96$ - - - - - 0.4750 - - - - - 47.50%

The risk of the last lender - - - - - 50% - 47.50% = 2.75%

CASE 6

Equity:1
Debt:2

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax (x)	Divergence $U=x-\bar{x}$	Probability p	UxP	U^2xP
3	600	102.5	497.5	248.75	156.41	1	156.41	24,464
2	400	102.5	297.5	148.75	56.41	3	169.23	9,546
1	200	102.5	97.5	48.75	-43.59	3	-130.77	5,700
0	0	102.5	-102.5	-102.5	-194.84	1	-194.84	37,962
Total					[illegible]	8	[illegible]	77,672

$$\bar{x} = \frac{738.75}{8} = 92.34$$

$$\sigma = \sqrt{\frac{77,672}{8}} = \sqrt{9,709} = 98.5$$

$$\bar{x} = 92.34 \times 20 = 1846.8$$

$$\sigma = 98.5 \times 20 = 1970$$

$$\bar{x} + E = 1,847 + 1,000 = 2,847$$

$$z = \frac{2,847 - 0}{1,970} = 1.440 - - - - - 0.4251 - - - - - 42.51\%$$

The risk of the second lender of - - - - 50% - 42.51% = 7.49%

CASE 7

Equity:1
Debt:3

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax (x)	Divergence $U=\frac{x-\bar{x}}{100}$	Probability p	UxP	U^2xP
4	800	157.5	642.5	321.25	2	1	2	4
3	600	157.5	442.5	221.25	1	4	4	4
2	400	157.5	242.5	121.25	0	6	0	0
1	200	157.5	42.5	21.25	-1	4	-4	4
0	0	157.5	-157.5	-157.5	-2.78	1	-2.78	7.7
Total					-0.78	16	-0.78	19.7

$\bar{x} = 121.25 - \frac{0.78}{16}(100) = 121.25 - 4.87 = 116.38$

$\sigma = 100\sqrt{\frac{19.7}{16}} = 100\sqrt{1.23} = 100 \times 1.1 = 110$

$\bar{x} = 116.38 \times 20 = 2327.6$

$\sigma = 110 \times 20 = 2200$

$\bar{x} + E = 3,328$

$z = \frac{3,328}{2,200} = 1.51$ - - - - - .4345 - - - - - 43.45%

The risk of the third lender - - - - - 50% - 43.45% = 6.55%

CASE 8

Equity:1
Debt:9

Number of success	Earning before interest	Interest	Net profit before tax	Net profit after tax (x)	Divergence $U = \frac{x-\bar{x}}{100}$	Probability p	UxP	U^2xP
10	2000	540	1460	730	5	1	5	25
9	1800	540	1260	630	4	10	40	160
8	1600	540	1060	530	3	45	135	405
7	1400	540	860	430	2	120	240	480
6	1200	540	660	330	1	210	210	210
5	1000	540	460	230	0	252	0	0
4	800	540	260	130	-1	210	-210	210
3	600	540	60	30	-2	120	-240	480
2	400	540	-140	-140	-3.7	45	-166.5	616.05
1	200	540	-340	-340	-5.7	10	-57	324.9
0	0	540	-540	-540	-7.7	1	-7.7	59.29
Total					-5.1	1024	-51.2	2970.24

$$\bar{x} = 230 - \frac{51.2}{1024}(100) = 230 - 5 = 225$$

$$\sigma = 100\sqrt{\frac{2970 - (51/1024)^2}{1024}} = 100\sqrt{\frac{2970}{1024}} =$$

$$100\sqrt{29} = 100 \times 1.7 = 170$$

$$\bar{x} = 225 \times 20 = 4500$$

$$\sigma = 170 \times 20 = 3400$$

$$\bar{x} + E = 4{,}500 + 1{,}000 = 5{,}500$$

$$z = \frac{5{,}500 - 0}{3{,}400} = 1.617 \text{ - - - - - } 0.4463 \text{ - - - - - } 44.63\%$$

The risk of the last lender - - - - -50% - 44.63% = 5.37%

TABLE 11

THE CHANGING RISK SITUATION BY THE CHANGE IN THE RATE OF INTEREST

Number of Investments	Number of Debts	Section I Situation		Section II Situation (arbitrary changing interest situation)	
		Probability of default	Interest rate	Interest rate	Probability of default
1	0	--	--	--	--
2	1	6.94	5.0	5.0	6.94
3	2	7.21	5.2	5.25	6.94
4	3	6.40	5.0	5.50	6.55
5	4	5.16	5.0	5.75	--
6	5	--	--	6.00	--
7	6	--	--	6.25	--
8	7	--	--	6.50	--
9	8	--	--	6.75	--
10	9	2.75	5.0	7.00	5.37

Section I Situation: The interest rate is determined by the risk rate.

Section II Situation: The interest rate is determined arbitrarily.

Chapter IV

SUMMARY

éral, a model was used in which additional financing was ed alternatively by debt and equity. This is in contrast to the onventional model which assumes that the total amount of capital is fixed, with only the debt-equity ratio changing. Using the new model an attempt was made to answer the pertinent question: how should the additional investments be financed? The solutions are summarized as follows: (1) under diversification, risk is decreased significantly by increasing total capital by either equity or debt financing, i.e., the diversification effect is always positive; (2) profitability cannot be increased by diversification--the diversification effect is limited to decreasing risk; (3) when total capital is increased by equity financing, the expected rate of return is not improved, this change serves only to decrease risk; (4) when the total capital is increased by debt financing the expected rate of return is improved and the risk decreases; (5) when debt and equity financing are compared through diversification, debt financing enables one to reach a higher investment indifference curve (higher level of satisfaction as determined by the combination of risk and expected rate of return). In other words, debt financing activates a leverage effect <u>and</u> a diversification effect, while equity financing yields only a diversification effect; (6) by increasing the total fixed capital, the short-term risk is decreased significantly by increased depreciation allowances; and (7) in certain diversified investment situations the lender's risk goes down as the total amount of debt capital increases. In this case there are no special reasons which justify imposing higher capital charges as the debt proportion in total financing increases. In other words, the assumed constant rate of interest appears to be a rather reasonable assumption within the framework of this model.

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